AN INVESTIGATION OF LEARNING CURVES AND THEIR USE IN SIMULATION

THESIS

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Jennie H. Lommel

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Abstract

In 1995, the C-17 Factory Simulation Model (FSM) was developed for the C-17 System Program Office at Aeronautical Systems Center (ASC), Wright-Patterson AFB, Ohio. Designed to enable analysts to address "what-if" questions about the resources required to build future aircraft, the FSM is based on learning curve models that are used to both portray and simulate future aircraft production.

In this thesis, we examine and develop alternate learning curve models that also utilize a small amount of initial production data (about 20 observations) to portray the relationship between the number of aircraft built and the amount of resources required to build them. The goal is to identify a model which not only provides a good fit and forecast based on a small amount of data but is also intuitive and reasonably simple to apply. In addition to examining variations on the Log-Linear Learning Curve model, we propose and evaluate the use of Box and Jenkins Autoregressive Moving Average (ARMA) models for modeling the effects of learning.

These models are exercised in fitting simulated log-linear data, as well as in fitting and forecasting historical F-102 manufacturing data and notional C-17 manufacturing data. The results are somewhat inconclusive since they do not identify any one model as the best for all data sets. They do, however, suggest that ARMA models are a very promising alternative to the standard log-linear learning curve.

The thesis concludes with an examination of the effects of explicitly accounting for uncertainty in parameter estimation when simulating future performance based on the

traditional log-linear learning curve model. The results show that the approach employed in the FSM is viable even though it does <u>not</u> directly account for this uncertainty.

AN INVESTIGATION OF LEARNING CURVES AND THEIR USE IN SIMULATION

1. Motivation for the Study

1-1. Background

One of the fundamental quantities of interest when estimating the life cycle costs of weapon systems such as the C-17, is an estimate of the resources required to build a given number of systems. This estimate provides the basis upon which many other costs and performance measures can be calculated. Given these, reasonably accurate answers can be given to questions such as, "How will the utilization of resources such as manpower or tooling be affected by changes in the 'buy profile' for the weapon system?" or "How will delivery dates be affected by changes in the assembly process?" and other "what if?" questions proposed by Congress and senior management regarding the effects of various changes in planning and production strategies. Models which can help to answer these kinds of questions are invaluable tools in the estimation of life cycle costs, but their development requires serious scrutiny and analysis if they are to provide useful information.

In many production, assembly, or maintenance operations, the amount of time required to complete a task tends to decrease each time the task is undertaken. A common approach in modeling this phenomenon is to use a *learning curve* in which the number of hours required to complete the task is modeled as a function of the number of units completed to date. This long-term relationship is often estimated on the basis of initial production data using straightforward regression procedures. The resulting fitted relationship then provides a forecast of the *expected* number of hours that will be required for the task to be performed in future operations .

When simulating such production processes, one needs not only to portray the *expected* number of hours required in future production, but also how those hours can be expected to vary about their mean. For example, in the C-17 Factory Simulation Model (FSM), which is a large-scale model of the C-17 assembly process recently developed for the C-17 System Program Office (SPO), this was handled by treating the fitted regression equation (based on data collected during the assembly of the first 20 aircraft) as if it provided a *perfect* forecast of the mean time required to complete a task in the future. Once this relationship was determined, random errors were generated about the fitted curve to simulate how actual performance might vary in the future. The validity of this procedure is somewhat in doubt, however.

An important source of doubt centers on how well this strategy can be expected to model future performance. This is an especially important question because the underlying learning curves have been estimated on the basis of a limited amount of data. The approach used in the C-17 FSM does not take into account the uncertainty surrounding the mathematical form of the learning curve model nor the values of the parameters within that model. For example, might the relationship be more accurately described with of an equation of a different form? Alternately, how (if at all) should predictions based on the fitted relationship account for one's uncertainty about the values of the parameters which specify that relationship? Should the simulation of the production of future aircraft account for this uncertainty? Further analysis of the appropriateness of this strategy and an investigation into possible alternative strategies is clearly warranted.

1-2. Problem Statement

An analysis of the current methods used in the C-17 FSM and, potentially, the development of improved models and methods for modeling and simulating learning curve relationships will enable analysts to provide better answers to the kinds of questions which are

asked in the acquisition process, and allow for better planning. This, in turn, will help to minimize (or *avoid* for the most part!) future cost and time overruns and allow for a more efficient allocation of resources.

1-3. Approach

In this thesis, we provide an analysis and assessment of some of the specific methods used to model and simulate learning curve relationships as implemented in the C-17 FSM. We begin with a thorough literature review to discover what is known about learning curves and to assess what new thinking must be done. The review covers a variety of learning curve models and the use of learning curves to simulate future performance. We next assess the adequacy of the approach used in the C-17 FSM and evaluate other possible approaches in an effort to determine what learning curve models might be most appropriate for this application.

We also propose a new approach to modeling and simulating learning using autoegressive [AR(p)] models and AutoRegressive Moving Average [ARMA(p,q)] models. Since we generally do not expect to have much data upon which to fit such models, it is necessary to keep the number of parameters (p and q) small; hence, we focus on AR(1) and ARMA(1,1) models.

We then attempt to make a formal comparison of the models examined, comparing them to each other, to data similar to that used in developing the C-17 FSM, and to historical data from the F-102 manufacturing program. These comparisons enable us to draw some basic conclusions and to make recommendations for future studies in this area. We conclude with an examination of the effects of explicitly accounting for uncertainty in parameter estimation in the traditional log-linear learning curve model.

2. Previous Work in Learning Curves

2-1. Definition of Learning Curve

A learning curve, also known as a progress, improvement, or experience curve, is a graphical or mathematical representation of how the requirement for resources is reduced as the production of a product or service is repeated. The learning curve concept may be used to predict production costs from the known costs of producing a product or service, the future service time from a history of service times, or the time required to build the n^{th} aircraft from a history of times spent building previous copies of the aircraft. The term “learning” is used in this thesis rather than “progress,” “improvement,” or “experience” because of its common use in the United States Air Force and most of the literature. “Learning,” as used here, includes worker learning, management innovations, engineering changes, and work simplification (Orsini, 1970:pgs 2:3).

The aircraft industry was the first to recognize the predictive value of learning curves. The earliest known work on the learning curve phenomenon was done by T.P. Wright who stated in his 1936 article, “Factors Affecting the Cost of Airplanes,” that he “started his studies of the variation of cost with quantity in 1922.” (Wright, 1936:122). He hypothesized that the cumulative average labor cost for any quantity of airplanes produced decreases by a constant amount as the quantity of airplanes is doubled. To calculate the cumulative average cost, we utilize equation (2-1).

$$Z_X = \frac{1}{X}\sum_{i=1}^{X} Y_X \tag{2-1}$$

where,

X is the unit number, and

Y_X is the number of hours (cost) for unit X.

As an example of an 80% learning curve, if it takes 100,000 hours of labor to produce Ship #1, it would take an average of 80,000 hours to produce Ships #1 and #2 (or 60,000 hours for Ship #2 alone), an average of 64,000 hours for the first four ships, 51,200 hours for the first eight ships, and so on.

In the example above, it is important to stress that the numbers 100,000 and 80,000 and 64,000, etc., are cumulative averages; they are the average costs of producing the first, first two, ans first four aircraft, respectively. The number describing the cost of Ship #2 alone (60,000) is known as its marginal or unit cost; it is the cost of producing the second aircraft alone. Most of the data considered in this thesis is marginal or unit cost data.

Who or what is doing the learning? In organizations, the learning curve describes the improvement in either individual productivity or organizational productivity. Individual learning is improvement that results when people repeat a process and gain skill or efficiency from their own experience. Organizational learning results from practice as well, but also comes from changes in administration, equipment, and product design.

Learning rates in organizations differ in their performance for a number of reasons. One factor affecting this performance, and hence affecting the learning rate, is the *volume* of the output; all other things being equal, the firm that has the higher cumulative output should have the lower cost. Another factor which weighs heavily is the *rate* of output; studies have shown that recent experience has much more effect in reducing cost than more distant experience. As a result, if we compare two companies

with the same cumulative output, the firm with the higher rate should have a lower cost curve because its experience is more recent. In organizations, both kinds of learning occur simultaneously but are most frequently modeled with a single learning curve.

2-2. Terminology

Before starting a discussion of the various forms of learning curves (Section 2-3, below) it is imperative that the terminology used throughout the remainder of this work be clearly defined. This is important because the terminology is not consistent between references. Some of the definitions (those annotated with an asterisk, *), have been taken directly from a thesis by Orsini (Orsini, 1970:pgs 2:3), the others have been gathered from other sources.

(1) <u>direct man-hours</u> - These are the hours expended to manufacture a unit of output. In the airframe industry, these hours consist of fabrication, assembly, production flight, and other production work associated with the basic aircraft (Orsini, 1970:pgs 2:3).

(2) <u>direct man-hours for Unit One</u> - The total direct labor hours expended to complete the first operable unit (Orsini, 1970:pgs 2:3).

(3) <u>learning rate</u> - A per cent figure which determines the number of direct man-hours required for each doubled production quantity in relation to the previous doubled quantity. For example, if a learning curve has an eighty percent learning rate, the direct man-hours required to produce unit 2 will be eighty percent the number required to produce unit one; the direct man-hours required to produce unit four will be eighty percent the number required to produce unit two; the direct man-hours required to produce unit eight will be eighty percent the number required to produce unit four; etc.

(4) cost - Refers to the quantity of resource required to perform a given activity. The cost could be in terms of dollars, direct man-hours, or other resource.

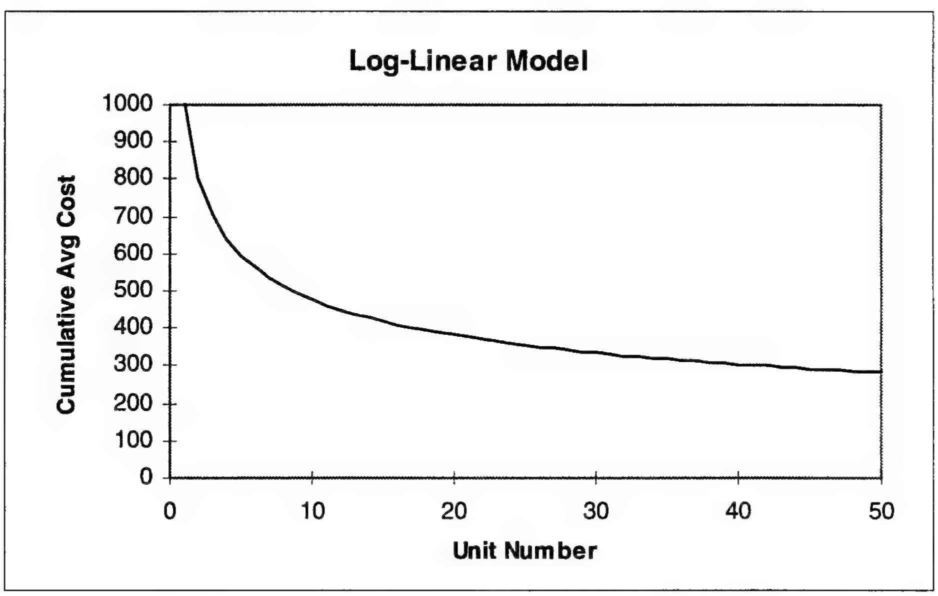
(5) cumulative average cost curve - A curve representing the cumulative average cost as the total number of units increases.

(6) unit marginal cost curve - A curve representing the average cost of each unit as the total number of units increases.

2-3. Geometric Versions of the Learning Curve

The Log-Linear (Crawford) Model. The most useful and most widely used learning curve model, the log-linear or constant percentage model, was first introduced in 1944 (Smith, 1989). This model, which states that the improvement in productivity is fairly constant as output increases, is depicted in Figure 2-1.

Figure 2-1 Example of a Log-Linear Model



The equation for this model is

$$Y \approx aX^{b} \qquad (2\text{-}2)$$

where

Y is the cumulative average amount of resources, or 'cost,' of producing the first X units,

X is the unit number,

a is the 'cost' of producing the first unit, and

b is a constant which determines the learning rate.

The learning percentage (learning rate), ρ, can be determined via

$$\rho = 2^b. \tag{2-3}$$

Equivalently, for a given learning percentage, b can be set by specifying

$$b = ln(\rho)/ln(2). \tag{2-4}$$

In order to estimate the parameters, a and b, of the model given above, a linear regression using a logarithmic transformation is conducted. The resulting model is

$$ln(Y) = ln(a) + b*ln(X). \tag{2-5}$$

According to Forsythe et al, there are some distinct disadvantages which arise from taking this standard approach. First of all, the large variability associated with the early observations will skew the estimated parameters which will then be dominated by the first few observations. Second, the per unit cost predicted by a log-linear learning curve will converge to zero for sufficiently large volume. If convergence to zero occurs within a realizable volume, the log-linear model produces unrealistic forecasts.

Both the problem of early observations skewing the estimated parameters, and the fact that the log-linear learning curve will converge to zero for sufficiently large volume are important considerations since they are far more pronounced when applied to unit

costs as opposed to cumulative average costs. This is especially important to us because the C-17 FSM implicitly applies the log-linear model to unit costs.

Pegel's Exponential Function. In an effort to develop more realistic forms of the learning curve, some authors have proposed models which are based on exponential functions. One of these authors, Pegel, proposed an algebraic exponential function to complement the power function (log-linear) model (Smith, 1989). Pegel's model gives the marginal cost per unit for the X^{th} unit as

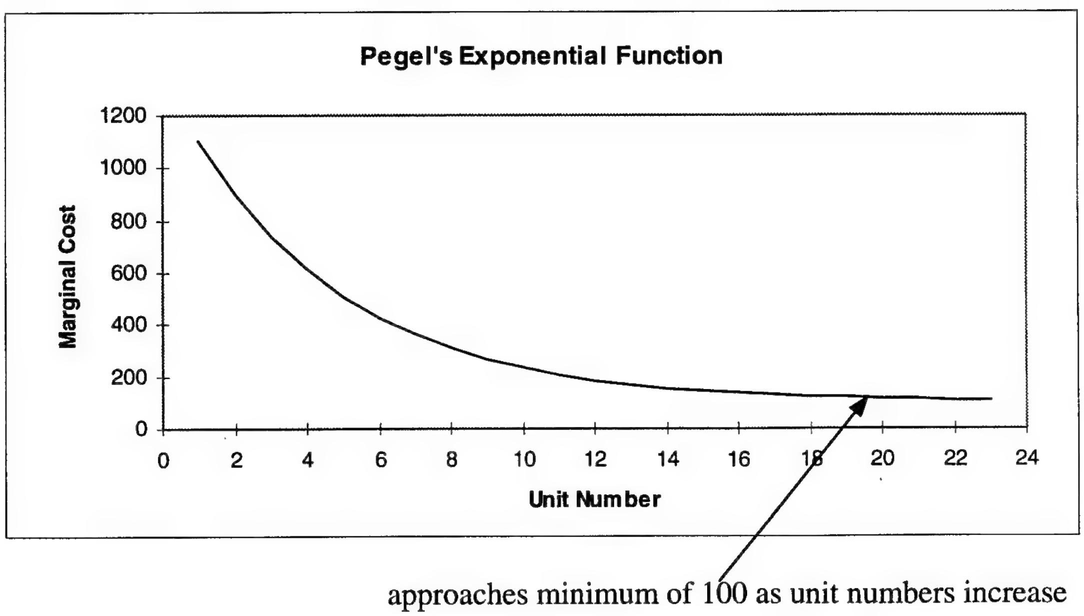
$$MC(X) = \alpha\, a^{X-1} + \beta \tag{2-6}$$

where

α, a, and β are empirically based parameters.

Using $\alpha = 1000$, $a = .8$, and $\beta = 100$, the curve appears as shown in Figure 2-2.

Figure 2-2 Pegel's Exponential Function



Note that, as the number of units increases, the marginal cost approaches a minimum value of 100 which is given by the value of β. This model may be integrated across X to

find the total cost up to the X^{th} unit in terms of the marginal cost per unit. It may also be algebraically manipulated to give the average cost of the first X units in marginal cost index terms. The biggest advantage in Pegel's model is that where the basic power function shows *both* the average and marginal costs decreasing with increasing output, Pegel's exponential function shows that the marginal cost becomes constant after a certain number of units is produced (Belkaoui, 1986:pgs 8:9).

Forsythe C_{min} Approach. A similar approach which addresses and includes the constant nature of marginal cost mentioned above is proposed by Forsythe (Forsythe, Green, White, and Elmer, 1995: pgs 3:4). He says that it is possible to tailor specific model parameters to account for characteristics of the manufacturing process. For example, when the absolute minimum number of hours required to produce a unit is known or can be estimated, the basic log-linear model can be revised as shown below.

$$Y(X) = a\,X^b + C_{min} \qquad (2\text{-}7)$$

In this equation, C_{min} is the minimum time needed to produce a unit. As the log-linear part of this equation approaches zero with large values of X, the time required to build the Xth unit approaches C_{min}; this makes intuitive sense. This adjustment, like Pegel's Exponential Function, prevents the time to produce a unit from reaching the unrealistic value of zero as the quantity produced, X, gets large.

Levy's Adaptation Function. Another twist on the marginal cost theme is given by Levy's Adaptation Function (Levy, 1966). Levy's function is useful for showing how a firm can adapt itself to the learning process and isolate the variables which influence learning. His function is given below.

$$MC = [1/\beta - (1/\beta - X^b/a)*C^{-CX}]^{-1} \tag{2-8}$$

where

MC is the marginal cost,

a and b are parameters analogous to the power function parameters,

C is analogous to Cmin, and

β is the production index for the first unit.

The parameter, C, serves to flatten the curve for large values of X. So, just as Pegel and Forsythe propose, the learning function reaches a plateau and does not continue to decrease (or increase) the way the log-linear function does (Belkaoui, 1986: pg 10).

The Stanford-B Model. One of the most well-known learning curve models is the Stanford-B Model. Like the models described above, this model came into existance because the log-linear model doesn't always provide the best fit to activity/time data. The Stanford-B formula, also known as the learning formula with the B-factor (Summers and Welsch, 1970: pgs 45:50), is given by the following equation.

$$Y = a(X+B)^n \tag{2-9}$$

where

Y is direct man-hours required for cumulative unit number X,

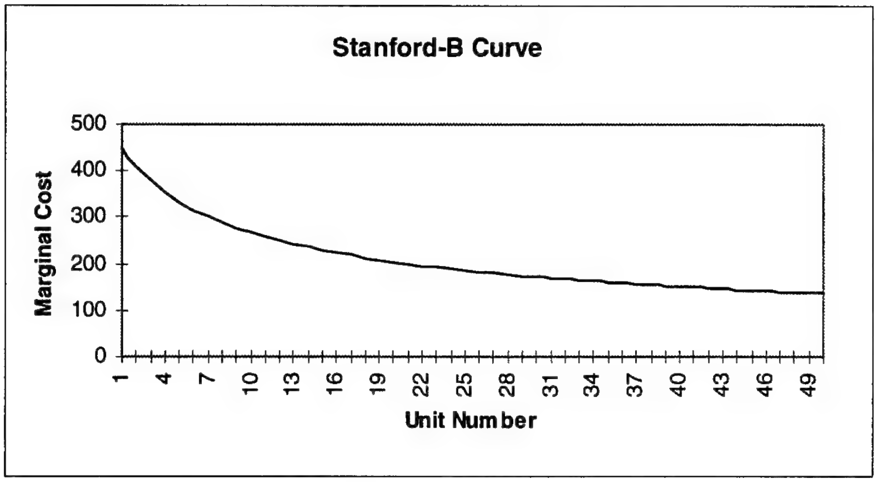
a is a constant which is equivalent to the cost of the first unit when B=0,

n is the exponent which describes the slope of the asymptote (-0.5 is typical),

B is a constant which may be expressed as the number of units theoretically produced prior to the first unit acceptance. (B is typically between 0 and 10 with 4 being a common value).

Given the 'typical' values above, the Stanford-B curve is depicted in Figure 2-3.

Figure 2-3 Stanford-B Curve



The main feature of the model is the B factor which measures variations in design or other complexities which management cannot control through engineering or retooling. Another way to think of this model is that it accounts for previous learning by using the B factor as a scale of displacement.

To illustrate, Belkaoui (Belkaoui, 1986:pg 11) suggests we note that when n is set equal to -0.5, the Stanford-B model yields a unit learning curve equation as follows:

$$Y \approx \frac{a}{\sqrt{x+B}} \tag{2-10a}$$

If B is set to 0, this becomes

$$Y \approx ax^{-0.5}. \tag{2-10b}$$

Using the equation

$$\rho = 2^b = 2^{-0.5} = .707, \tag{2-10a}$$

we can see that this is equivalent to a 70.7 percent log-linear learning curve. Smith notes that the Stanford-B isn't used much in the aircraft industry anymore (Smith, 1989).

<u>DeJong's Learning Formula with an Incompressibility Factor.</u> Another form of the log-linear model, which is similar to the models proposed by Pegel, Forsythe, and Levy (in that the value of MC levels out after a large number of builds) is DeJong's Learning Formula with an Incompressibility Factor. This model takes into account the differing nature of manual and machine labor. DeJong refers to manual activity time as being compressible (subject to learning); he considers machine activity time to be incompressible. His model has the following form which consists of a variable part representing manual activity and a fixed part representing machine activity.

$$MC \approx a(\underbrace{M}_{\text{Fixed Part}} + \underbrace{\frac{1-M}{X^{n}}}_{\text{Variable Part}}) \qquad (2\text{-}11)$$

where

MC is the marginal time for the xth unit,

a and n are parameters analogous to the power function parameters, and

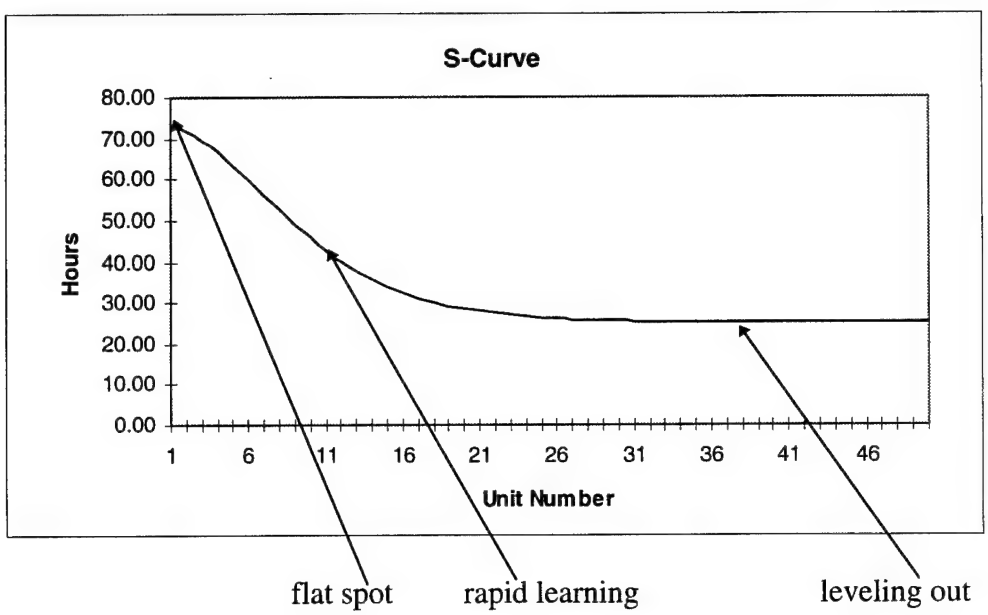
M is the factor of incompressibility.

The value of M varies between 0 and 1 and would be, approximately, .25 for manually dominated activities and .50 for machine-dominated activities. Note that for an activity which is 100% manual, M would be zero and DeJong's formula reduces to the log-linear model. For an activity which is 100% machine dominated, M equals 1 and MC is a constant. Since we really don't expect machines to be able to improve their performance this makes intuitive sense. Some learning curve prognosticators (Smith, 1989: pg 7) have suggested, however, that machines *can* 'learn' through the adjustment of machine parameters or production methods. Smith suggests that, "The DeJong model may not be

particularly useful in machine intensive industries, especially considering the model adds so much more complexity to learning curve mathematics and almost eliminates any intuitive understanding of real-life application to the typical factory worker or cost estimator." (Smith, 1989)

The S-Curve. The S-Curve is another curve which has been used to model the learning process seen in manufacturing. An S-type function can be represented by a composite function whose shape resembles the flattened horizontal S shown below.

Figure 2-4. The S-Curve



Note the flatness of the earliest part of the curve. This can be attributed to the fact that experiments are being made on the production process in an effort to find the best tooling and methods. The numerous mid-course changes made at this time preclude rapid improvement in the amount of time to build each unit. Once all the corrections are made to the toolings and methods, it is possible to obtain very rapid learning; this is shown in

the center part of the curve. Finally, once most of the learning has been done, the curve starts leveling out and approaches a minimum time required to build an individual unit.

As mentioned above, the S-Curve can be represented by a composite function. According to Belkaoui (Belkaoui, 1986:pg 14) we can constuct an S-function by using the Stanford-B curve to represent the early part of the curve and the DeJong curve to represent the latter part of the curve. In this case, the S-Curve function would look like:

$$MC = a[M + (1 - M)(X + B)^n] \qquad (2\text{-}12)$$

where (as before):

MC is the marginal time for the Xth unit,

a and n are parameters analogous to the power function parameters,

M is the factor of incompressibility, and

B is a constant which may be expressed as the number of units theoretically produced prior to the first unit acceptance. (B is between 0 and 10 with 4 being a typical value).

Like all models, S-shaped curves have their advantages and dissadvantages. These curves can model more complex learning curves which do not follow a simple exponential form. However, when we have initial production data, only S-shaped curves may not have sufficient data to accurately estimate long-term production.

Non-Linear Estimation. Thomas (Thomas, 1975) compared log-linear and nonlinear estimation techniques for the standard learning curve model. Each model assumes that the error is distributed normally under the logarithmic transformation and that it tends to be proportional to the value of the dependent variable and multiplicative in nature. The nonlinear estimation techniques, on the other hand, generally assume a

constant error term which is additive. Thomas found that the nonlinear estimation techniques were robust and performed well in estimating the parameters of the model with *either* error distribution; the log-linear model did not perform as well when the error was additive and constant in nature.

2-4. Summary

Over the course of the last eighty years, the learning curve has been used extensively as a management accounting tool; this type of curve can provide important information for decision making. In this chapter, we have presented the concept of the learning phenomenon and have gone on to briefly outline not only the theory behind the basic log-linear learning curve but some 'new' geometric versions as well.

The Crawford Model (or the basic log-linear learning curve model) states that the improvement in productivity is fairly constant as output increases. The main shortcoming of this model lies in the fact that as the number of units produced increases, the time/cost to produce them approaches the unrealistic value of zero. The Forsythe and Pegel models are more realistic than the Crawford since they include a minimum cost per unit which does not allow the unrealistic approach towards zero. The Stanford-B model attempts to account for prior learning by including a displacement or B factor.

The models mentioned in the preceding paragraph, seem to be worth further investigation since they are fairly simple and are intuitive for the most part. On the other hand, due to their complex nature, we will not devote further study to of DeJong's Learning Formula, and Levy's Adaptation Function within this thesis. Although the S-Curve has the ability to model more complex learning functions which do not follow the

simple exponential form, it may also be unsuitable due to its complexity. Further, the S-Curve model may require more than just initial production data if they are to accurately estimate long term production; initial production data is frequently the only data we have.

Variations on the basic learning curve have proliferated mainly because the log-linear model does not always provide a good fit or forecast for the data at hand. The next chapter explores a completely different class of models as an alternative method of predicting future aircraft build times; enter the ARIMA models.

3. Proposed ARMA Learning Curve Models

3-1. Introduction to ARMA Models

With the advent of widespread computer availability in organizations, the general and statistically based methods of time-series analysis know as Box-Jenkins or ARIMA processes have been developed and applied to forecasting (Makridakis, Wheelwright, and McGee, 1983). ARIMA, which is an abbreviation for autoregressive (AR) integrated (I) moving average (MA), describes a broad class of time-series models. Before we present a detailed discussion of the two basic ARIMA processes which are of interest in this study, we briefly define some of the terms used in the rest of this chapter and discuss why these processes are being explored.

The following definitions are based on discussions found in Makridakis, Wheelwright, and McGee (1988), Box and Jenkins (1976), and Mongomery, Johnson, and Gardiner (1990). A process is stationary if its statistical properties are independent of the particular time during which it is observed. Specifically, if the process underlying a time series is based on a constant mean and variance, then the time series is said to have a stationary mean and variance. If a process is stationary, the Box and Jenkins model used is generally an ARMA(p,q). As discussed in subsequent sections of this chapter.

A process is nonstationary if its statistical properties (especially its mean and variance) depend on the time during which it is observed. In other words, if the process underlying the time series does not have a constant mean and/or a constant variance, it is nonstationary. If a processes is non-stationary, the Box and Jenkins model generally used

is an ARIMA(p,d,q) with the differencing term, d, *not* equal to zero. In this thesis, attention will be focused on stationary models.

An ARIMA model is parsimonious if it uses as few parameters as possible in the model/data fitting process. For example, if we assume that an AR(1) process, which has one parameter, and an AR(2) process, which has two parameters, both provide reasonable fits to a particular data series, the concept of parsimony would have us choose the AR(1) model over the AR(2).

An autoregressive process is a form of regression where, instead of the dependent variable (the item to be forecast) being related to independent variables, it is related to past values of itself at varying time lags. Thus, an autoregressive model would express the observation at time t as a function of previous values of that time series. This matches up well with the logic behind learning curves since, if learning is indeed occurring, one would expect that the amount of resources required to produce a given unit would depend, or be related to, the amount of resources required to produce previous units.

A moving average process is a process in which the value of the time series at time t is influenced by a current error term and (possibly) weighted error terms from the past. The error terms of which we speak, are independent and identically distributed random noises or shocks that are generally uncontrollable or unpredictable.

Autoregressive/moving average (ARMA) schemes can be autoregressive (AR) in form, moving average (MA) in form, or a combination of the two (ARMA). In an ARMA model, the series to be forecast is expressed as a function of both previous values of the series (autoregressive terms) *and* previous random errors (the moving average terms).

In the next three sections I will discuss, in somewhat more detail, the nature and structure of three ARIMA processes, AR(p), MA(q), and ARMA(p,q), (more formally known as ARIMA(p,0,0), ARIMA(0,0,q), and ARIMA(p,0,q) processes respectively) which are of interest in this study. We'll also consider, the suitability of each model to simulate learning.

3-2. Autoregressive Processes

As stated above, autoregressive (AR) processes are a form of regression where the dependent variable is related to past values of *itself* at varying time lags. This might seem contrary to regression methods which attempt to forecast variations in some variable of interest, the dependent variable, on the basis of variations in a number of other factors, the independent variables. The general form of AR processes may be developed by starting with the basic causal or explanatory regression equation which has the form

$$Y = b_0 + b_1X_1 + b_2X_2 \ldots + b_kX_k + e \qquad (3\text{-}1)$$

where

Y is the dependent variable,

$X_1, X_2, \ldots, X_k$, are the independent variables,

$b_0, b_1, b_2, \ldots, b_k$ are the linear regression coefficients, and

e is an error term which is assumed to have

$$\mathrm{E}[e] = 0 \text{ and } \mathrm{Var}\,[e] = \sigma^2.$$

The independent variables, $X_1, X_2, \ldots, X_k$, can represent any factors such as the number of parts to be installed, the quality of parts, the type of aircraft to be built, while the dependent variable, Y, could represent the time it took to build an aircraft.

Some principles of regression can be applied to time series methods. We could, for example, allow the independent variable to represent previous values of Y, the time required to build the aircraft at previous times in the past, and suggest that the time it takes to build the t^{th} aircraft depends *not* on the number of parts to be installed or on the quality of the parts, but on the time it took to build the t-1^{st}, and even t-2^{nd} and t-3^{rd}, etc., aircraft in the past. We could define the dependent variable as

$$Y_t = \mu' + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \phi_3 Y_{t-3} + \phi_4 Y_{t-4} + \ .... + \phi_k Y_{t-k} + e_t \tag{3-2}$$

where

Y_t is the value of the dependent variable at time t,

$Y_{t-1}, Y_{t-2}, Y_{t-3}, Y_{t-4}, \ldots, Y_{t-k}$, are the 'independent' variables, which represent the prior values of the dependent variable,

$\mu', \phi_1, \phi_2, \ldots, \phi_k$ are the linear regression coefficients, and

e_t is an error term which is assumed to have

$$E[e_t] = 0 \text{ and } Var[e_t] = \sigma^2.$$

This is called the basic autoregressive (AR) form since the independent factors are simply time-lagged values of the dependent variable.

One of the AR(p) models which I explore as part of this thesis is the AR(1) model. The model takes the following form

$$Y_t = \mu' + \phi_1 \ Y_{t-1} + e_t \tag{3-3}$$

where the the variables are the same as defined in equation (3-2). Before plotting the AR(1) model given in equation (3-3), we'll define the difference between unconditional and conditional means.

If ϕ_1 is chosen between -1 and +1, the AR(1) process is stationary with mean

$$E(Y) = \mu = \frac{\mu'}{1-\phi_1}. \tag{3-4}$$

Since we make no assumption about previous values observed, this is referred to as the unconditional mean. The unconditional mean differs from the conditional mean in that the conditional mean takes previous values of the time series into account. For example, suppose for an AR(1) process, the value observed at time t-1 is y_{t-1}. Then the expected value at time t given this information is given by:

$$\begin{aligned} E[Y_t | Y_{t-1} = y_{t-1}] &= \\ &= E(\mu' + \phi_1 Y_{t-1} + \varepsilon_t | Y_{t-1} = y_{t-1}) \\ &= \mu' + \phi_1 y_{t-1} + E(\varepsilon_t) \\ &= \mu' + \phi_1 y_{t-1} \end{aligned} \tag{3-5}$$

where we assume that $E(\varepsilon_t) = 0$. The quantity in equation (3-5) is not necessarily equal to μ. In particular,

$$E(Y_2 | Y_1 = y_1) = \mu' + \phi_1 y$$

and similarly

$$\begin{aligned} E(Y_3 | Y_1 = y_1) &= E(\mu' + \phi_1 Y_2 + \varepsilon_2) \\ &= \mu' + \phi_1 E(Y_2 | Y_1 = y_1) + E(\varepsilon_2) \\ &= \mu' + \mu'\phi_1 + \phi_1^2 y_1 \\ &= \mu'(1+\phi_1) + \phi_1^2 y_1 \end{aligned}$$

In general, for $t \geq 2$,

$$E(Y_t | Y_1 = y_1) = \mu'(1 + \phi_1 + \phi_1^2 + \ldots + \phi_1^{t-2}) + \phi_1^{t-1} y_1 \tag{3-6}$$

This illustrates the fact that the initial conditions (the value observed at time 1) has decreasing effect on the long run provided that $-1 \leq \phi_1 \leq +1$. In fact, in the long run,

$$\lim_{t \to \infty} E(Y_t | Y_1 = y) = \mu$$

$$= \lim_{t \to \infty} \left[\mu' \sum_{i=1}^{t-2} \phi_1^i + \phi_1^{t-1} y_1 \right]$$

$$= \frac{\mu'}{1 - \phi_1} \tag{3-7}$$

provided $-1 \leq \phi 1 \leq +1$. The quantity given in equation (3-7) is the unconditional mean.

To see why the the unconditional mean is useful in simulating the effects of learning, suppose $\mu' = 25$, $\phi_1 = 0.75$ so that $\mu = \frac{\mu'}{1-\phi_1} = 100$. We can then generate simulated values of Y_t for $t = 2, \ldots, 50$ assuming that $Y_1 = 1000$ and the e_t's are uniformly distributed with $E(e_t) = 0$ and $Var(e_t) = 25$. The plot which results from using the above assumed values in equation (3-3) (the equation forAR(1)) is given in Figure (3-1)

Figure 3-1 Illustration of the AR(1) Model

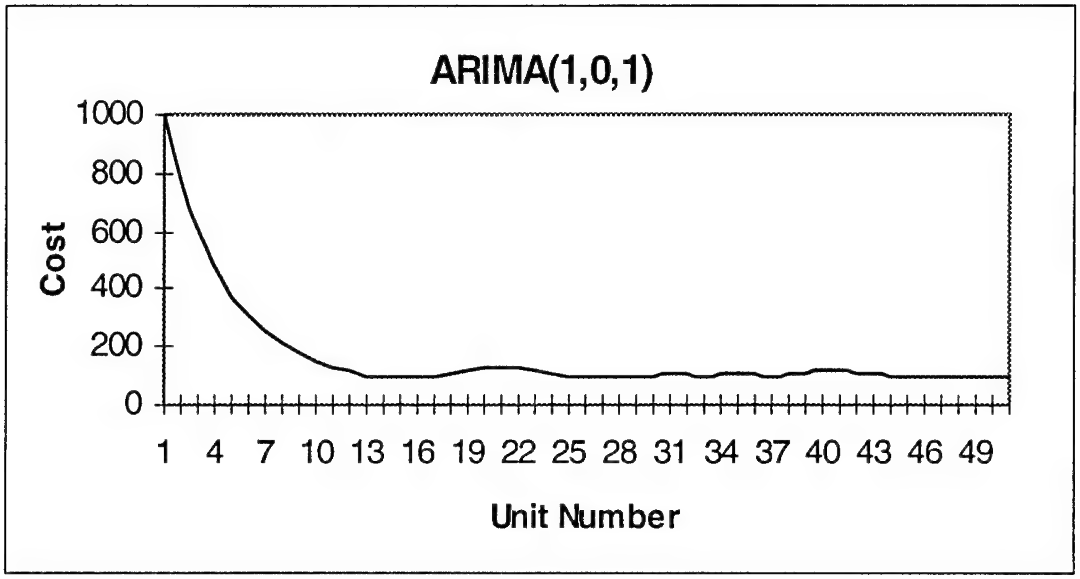
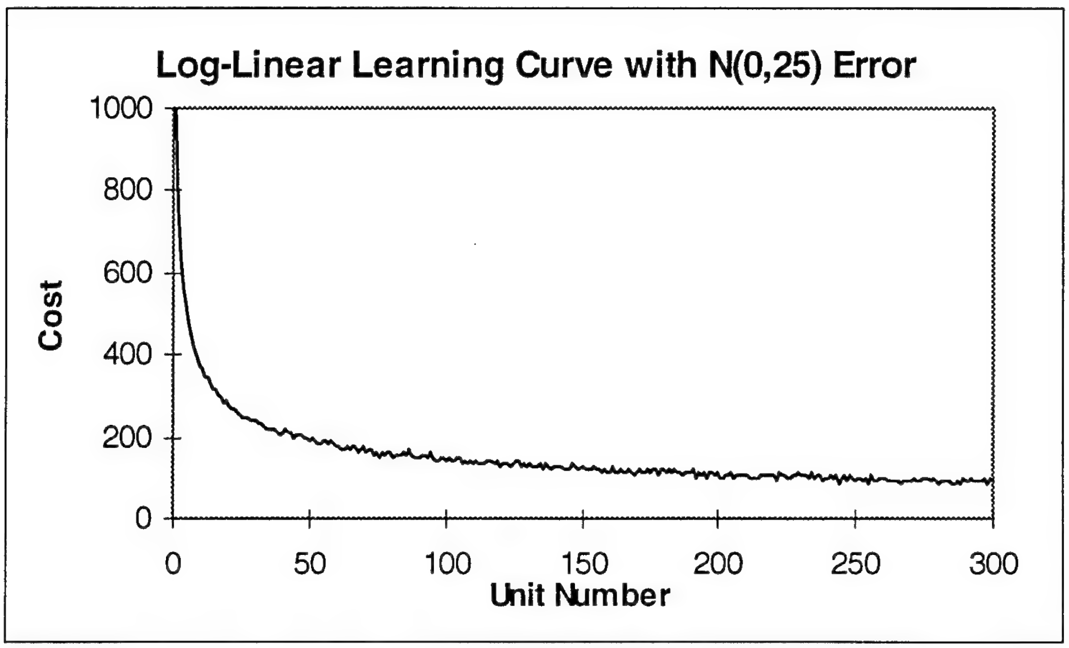


Figure 3-2 Illustration of the Log-Linear Learning Curve Model



In Figure 3-2, we have a plot of a typical learning curve for comparison. Note the similarity between the AR(1) curve and the typical learning curve. The main difference between the two curves is that in the long run, the AR(1) model approaches

$$E(Y) = \mu = \frac{\mu'}{1-\phi_1} = 100$$

while the learning curve continues to approach zero. As a result,the AR(p) model may actually prove to be more useful for predicting learning types of data.

3-3. Moving Average Processes

In a manner analogous to writing the basic regression equation (3-1) in terms of past values of the dependent variable to define the AR process given by equation (3-2), we define an MA process by writing the basic regression equation in terms of past error terms; in other words, we let the past error terms be the independent variables. The general equation for an MA model is

$$Y_t = \mu + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \theta_3 e_{t-3} + \ldots + \theta_k e_{t-k} + e_t \qquad (3\text{-}8)$$

where

Y_t is the value of the dependent variable at time t,

e_t, e_{t-1}, e_{t-2}, e_{t-3},... , e_{t-k}, are the 'independent' variables, which represent the time-lagged error terms. These error terms are defined as $e_t = X_t - Y_t$, $e_{t-1} = X_{t-1} - Y_{t-1}$, $e_{t-2} = X_{t-2} - Y_{t-2}$, and so on. X is the actual value while Y is the forecasted value.

μ, θ_1, θ_2, ... , θ_k are the linear regression coefficients.

One of the MA(q) models which we explore as part of this thesis is the MA(1) model. The model takes the following form

$$Y_t = \mu + e_t - \theta_1 e_{t-1}. \qquad (3\text{-}9)$$

To assess the usefulness of the MA(1) models for forecasting learning data we fix $Y_1 = 1000$, $\theta_1 = -0.75$, and assume the errors are ~ N(0,25); specifically,

$$E[e_t] = 0 \text{ and } Var[e_t] = \sigma^2 = 25.$$

Since we wish to fix $Y_1 = 1000$, this implies

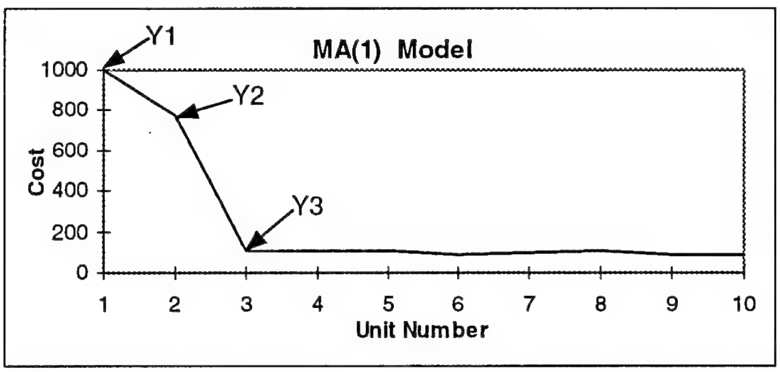
$$1000 = 100 + e_1 - \theta_1 e_0.$$

If we assume $e_0 = 0$, then we should have $e_1 = 900$. After time 1, we go on to compute ten more values of Y_t using equation (3-10).

$$Y_t = 100 + e_t - \theta_1 e_{t-1} \tag{3-10}$$

The plot of the resulting data is given in Figure (3-3).

Figure 3-3 Illustration of MA(1) Model



Observe that Y_1, the cost of unit 1, is 1000 (by design). The value of Y_2 is affected by the large error at unit 1. By unit 3, the effect of unit one on the cost seems to be almost completely gone. Since the effect of unit one dissipates so quickly, we feel that MA models will not be useful for forecasting learning type data.

3-4. AutoRegressive/Moving Average Processes

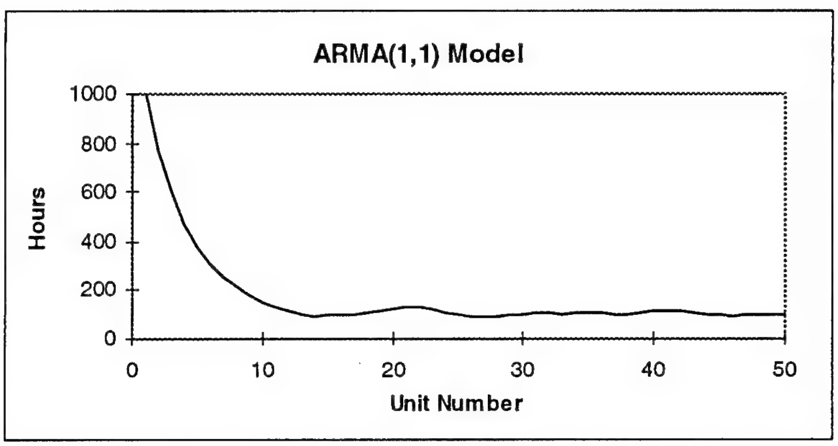
As stated in Section 1, ARMA schemes can be autoregressive (AR) in form, moving average (MA) in form, or the two can be effectively coupled to form a very general and useful class of time-series models called autoregressive/moving average (ARMA) processes. In the ARMA model, the series to be forecast is expressed as a

function of *both* the previous values of the series (the autoregressive terms) *and* the previous random errors (the moving average terms). The basic elements of AR and MA processes can be combined to produce a large variety of models. As part of this study we explore the ARMA(1,1) model which has the following form

$$Y_t = \underbrace{\phi_1 Y_{t-1}}_{AR(1)\ part} + \underbrace{\mu'}_{Constant} - \underbrace{\theta_1 e_{t-1} + e_t}_{MA(1)\ part} \tag{3-11}$$

Here, the dependent variable, Y_t, depends on one *previous* value of the dependent variable, Y_{t-1}, one previous error term, e_{t-1}, and a constant which adjusts the mean of the process. Under specific conditions on the θ*'s*, ARMA models are considered stationary in both the mean and the variance. A plot of a simulated ARMA(1,1) process is shown below.

Figure 3-4 Illustration of the ARIMA(1,0,1) Model



This particular model is calculated using ϕ_1 and θ_1 equal to .75, μ' equal to 25, and Y_1 equal to 1000. The initial value for error, ε_1, was taken as zero since the first unit cost was given with certainty. The remaining errors have an N(0,25) distribution. Note how the curve levels out after about twenty units to a mean of approximately 100; this is similar

to the curve calculated for the AR(1) model in Section 3-2. ARMA models appear to be well suited to forecasting learning type data.

3-5. ARIMA Models for Learning

As discussed earlier, a major criticism of the basic log-linear learning curve is that it does not completely level out in the long run; it continues to approach zero through successive unit builds. We find this pleasing, 'though unrealistic, because it implies that if we build enough aircraft, it eventually will take *no time at all* to build them!! In Chapter 2, this problem was addressed through the use of modified learning curve models such as the Stanford B Model and Pegel's Exponential Function which both provided curves that level out in the long run. In this chapter, however, we have presented examples of ARMA models that exhibit a similar pattern of a dramatic initial decline followed by a leveling out to a long-run mean. This suggests that stationary ARMA models might be useful for modeling the build history of an aircraft manufacturing process.

In the next chapter, we attempt to fit ARMA models to real and simulated sets of learning data, and compare the results with those of fitting standard learning curve models to the same data. In the next section, however, we first exploit a commercial forecasting software package, FORECAST PRO, to fit various ARMA models to the first 50 observations from a simulated log-linear learning curve. The purpose of this exercise is simply to assess how well stationary ARMA models can represent traditional log-linear learning curves, especially in the short-run. (In the long-run, we know that a stationary ARMA model converges to its unconditional mean while the log-linear learning curve converges to zero.)

3-6. Fitting ARMA Models to a Log-Linear Learning Curve

To represent a log-linear learning curve, we simulate the first 50 values from a model of the form

$$Y = aX^{b} + e \tag{3-12}$$

where (as presented in Chapter 2)

Y is the number of hours, or 'cost,' of producing the X^{th} unit,

X is the unit number,

a is the 'cost' of producing the first unit,

b is a constant which determines the learning rate, and

e is error uniformly distributed (1% above and 1% below the expected value of Y for each X.

Arbitrarily, and for purposes of illustration, we let the parameter, *a*, the cost of producing the first unit, be equal to 1000. We additionally assume the learning rate, ρ, to equal eighty percent (0.8), and then calculate the value of *b* using equation (2-3) as follows:

$$b = ln(\rho)/ln(2) = ln(0.8)/ln(2) = -.322. \tag{2-3}$$

For our value of ρ, b turns out to equal - 0.322. The final model which we use to generate data for units 1 through 50, takes on the following form:

$$Y = 1000\, X^{-0.322} + \mathrm{e} \tag{3-13}$$

To generate the data, we start by using the spreadsheet software package, EXCEL 5.0, by Microsoft Corporation, to generate the values of *Y* for units 1 through 50 by using equation (3-13). Since the equation used to calculate the data has a random component to it (error is uniformly distributed about the mean) the simulated data does also. For this reason, we can't just take one sample data set, analyze it and draw conclusions; we need

additional data sets to analyze. For our purposes, we generate eight sets of simulated data. A sample of this data is given in Table 3-1.

On the other hand, in hindsight, the variance of the random error terms that we have simulated is so small that we essentially are fitting models to the first 50 values <u>expected</u> from the log-linear model. Thus, our results provide a measure of how well the fitted models reproduce the expected behavior of a log-linear learning curve.

Table 3-1 Fifty Unit EXCEL-Simulated Log-Linear Data Set

1	1069.95	11	475.01	21	372.24	31	340.78	41	322.60
2	800.81	12	461.84	22	346.71	32	337.70	42	306.84
3	766.02	13	476.05	23	386.67	33	337.56	43	278.35
4	622.79	14	387.03	24	340.10	34	324.59	44	270.31
5	577.24	15	376.43	25	330.89	35	312.11	45	278.97
6	586.41	16	421.72	26	325.96	36	303.20	46	268.07
7	530.17	17	377.53	27	343.74	37	308.34	47	266.63
8	464.78	18	357.46	28	336.64	38	281.67	48	286.57
9	521.68	19	382.15	29	321.94	39	310.45	49	277.83
10	513.01	20	416.99	30	355.04	40	332.18	50	263.90

As a preliminary investigation into the potential of candidate models to fit and forecast log-linear types of data, we use FORECAST-PRO to fit selected ARMA(p,q) models to each of the eight data sets. The models we fit are AR(1), AR(2), AR(3), AR(4), MA(1), MA(2), MA(3), MA(4), ARMA(1,1), ARMA(1,2), ARMA(2,1), and ARMA(2,2). Since we are assuming that the process underlying the actual aircraft build time-series is stationary (which, however, is not strictly true for the log-linear data), we choose *not* to fit any ARIMA($p,d,q|d \neq 0$) to the simulated data. While using Forecast-

Pro, curiosity also dictates that we run its Expert Data Exploration routine, and also fit various Exponential Smoothing models to the data.

Using Forecast-Pro, we fit the selected models to the simulated log-linear data. The output standard diagnostics (including R^2, Adjusted R^2, Durbin-Watson and Ljung-Box test statistcis, MAPE[1], MAD[2], BIC[3], RMSE[4], and the Forecast Errors) from Forecast-Pro are given in Appendix A. In addition to the standard diagnostics, Appendix A contains plots of the data (the models for only one of the eight repetitions is shown), including the fitted curves, forecasts with confidence intervals, and model parameter estimations along with their associated Standard Errors, t-Statistics, and Significance levels. This data is consolidated into one table, Table A-1. The data is reduced to essential measures of model appropriateness (averaged across all eight repetitions) which is included, below, in Table 3-2. Tables similar to Table 3-2 are also given in Appendix A for each of the eight repetitions; they are given as Tables A-2 through A-9.

[1] MAPE - Mean Absolute Percent Error
[2] MAD - Mean Absolute Deviation
[3] BIC - Bayesian Information Criterion
[4] RMSE - Root Mean Square Error

Table 3-2 Model Measures of Appropriateness Summary

Key
Four Best Results in Category
Better Than Average
Worse Than Average
Overall best in Category

Simulation Model	RSQR	ADJRSQ	MAPE	MAD	RMSE
Simple Exp. Smoothing	0.8905	0.8908	0.0639	27.4850	45.4950
SMA(1)	0.8348	0.8394	0.1519	27.9463	45.9000
AR(1)	0.8976	0.8988	0.0639	27.3638	44.9650
AR(2)	0.9126	0.9105	0.0621	26.2300	39.8150
AR(3)	0.9243	0.9211	0.0619	25.4559	37.3200
AR(4)	0.9350	0.9308	0.0607	24.8680	34.7200
MA(1)	0.7077	0.7077	0.1377	54.4250	76.3150
MA(2)	0.8619	0.8590	0.1019	39.9700	51.9988
MA(3)	0.9027	0.8986	0.0821	32.4688	42.4975
MA(4)	0.9223	0.9173	0.1860	29.8850	37.5200
ARMA(1,1)	0.9066	0.9047	0.0641	27.1563	42.1275
ARMA(2,1)	0.9161	0.9125	0.0626	26.0114	38.7388
ARMA(1,2)	0.9381	0.9224	0.0615	25.2964	36.6213
ARMA(2,2)	0.9261	0.9217	0.0610	24.9766	36.1988
AR Average	0.9174	0.9153	**0.0621**	25.9794	39.2050
MA Average	0.8487	0.8456	0.1269	39.1872	52.0828
ARMA Average	**0.9217**	**0.9153**	0.0623	**25.8602**	**38.4216**
sum	12.4763	12.4352	1.2213	419.5383	610.2325
average	0.8912	0.8882	0.0872	29.9670	43.5880

Observing the data contained in Table 3-2, we note the following:

(1) Starting at the top of Table 3-2, we see that Simple Exponential Smoothing, and SMA(1)[5], turn in a relatively poor performance. The statistics for simple exponential smoothing are below average[6] in the categories of RSQR, and RMSE, but are above

[5] SMA - Single Moving Average, in which the forecast for time t+1 is simply the observation at time t

[6] The statistics which are calculated (and which are displayed in Table 3-2) for the various models under study in this section cannot all be compared directlly. For example, the RSQR and ADJRSQ statistics are considered better if their magnitude is larger. The MAPE, MAD, and RMSE, being measures of error are considered to be better if their magnitudes are smaller. So, for purposes of clarity, we refer to statistics as being better or worse than average. A better than average RSQR statistic is a statistic which is larger

average in the categories of MAPE, ADJRSQ, and MAD. Four out of five of the statistics for SMA(1) are below average; the only above average statistic is in the category of MAD.

(2) The AR models seem to do quite well. AR(1) and AR(2) have above average statistics in all but one category; AR(1) is below average in RMSE. AR(3) and AR(4) do remarkably well in that their statistics are among the best four statistics in every single category.

(3) As expected, the MA models have generally the poorest results in the investigation. MA(1) and MA(2) have below average scores in every single category. The MA(3) and MA(4) models do better; each of them are above average in four of the five categories; MA(3) is below average in the MAD category while MA(4) is below average in the MAPE category.

(4) Like the AR models, all of the ARMA models are above average in all categories; ARMA(1,2) and ARMA(2,2) have statistics which are among the best four in every single category.

The overall results seem to suggest that the Simple Exponential Smoothing model and the Simple Moving Average model do a mediocre job of fitting the simulated log-linear learning curve data. Consider the AR, MA, and ARMA averages given at the bottom of the table; they summarize the over-all findings quite well. The AR models, overall, are better than average in all categories, are best in the category of MAPE, and are tied for best with the ARMA models in the category of ADJRSQ. Overall, the MA models turn in a below average statistic in every single category. Finally, overall, the

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than the average RSQR, while a better than average MAPE statistic is lower than the average MAPE

ARMA models do very well in fitting the data. They had the best average statistic in all categories but one; they were simply above average in the category of MAPE.

What we've established is that the AR and ARMA models generally seem well suited to fitting the log-linear data despite the fact that the true log-linear model has a nonstationary mean. Observation of the statistics reveals, however, that the statistics differ very little from one model to another within each category. The fact that one is better than another is just a puff of air. The only serious difference between statistics from one model to another occurs in the MA(1) and MA(2) models which seem to have much worse statistics than the other models.

Now that this analysis is complete, we move on to a forum which provides equal treatment to *all* potential models[7] including the modified learning curve models discussed in Chapter 2. This is the topic on which we concentrate our energy in Chapter 4.

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statistic.

[7] Forecast-Pro's repertoire of models does not include models based on the learning-curve.

4. A Comprehensive Model Comparison

4-1. Introduction

To be useful in program management, a model must be accurate. This raises a very important question, "How do we pick a model from among the models we've been considering thus far, that will provide reasonably accurate predictions of future build times?"

To help focus our sights, our next effort is organized into three main stages which are discussed in detail in Sections 4-2, 4-3, and 4-4, respectively. In each stage we fit the candidate models to a particular set of data, using the spreadsheet program EXCEL to estimate the various model parameters. The purpose of using EXCEL is to put each model into an environment where it may be compared with the other potential models on an equal basis. (Forecast Pro is good for helping choose some promising *ARIMA* models but since it doesn't have a facility to fit learning curve models, it really can't be used to complete the analysis; there's just no common footing.)

The first stage, discussed in Section 4-2, uses a simulated log-linear data base to test the potential models' ability to fit a log-linear curve. The second stage, discussed in Section 4-3, uses data obtained from the F-102 production program to

1. test the potential models' ability to *fit* the complete data set, and to
2. test, using a hold-out set[1], the potential models' ability to *forecast* future build times.

[1] 'Hold out set,' is a reference to the method of using a portion (x out of n observations) of a given set of data to develop a model. The resulting model is then used to forecast the next n-x values in the series; the

Finally, the third phase, discussed in Section 4-4, sets out to forecast future build times in the C-17 program.

4-2. Fitting Simulated Log-Linear Data

In this part of the investigation, we take a closer look at each of the most promising Log-Linear and ARMA models identified in Chapters 2 and 3. Each of the models is used to fit a data base consisting of a 50-unit sequence of simulated log-linear data.

First, EXCEL is used to generate fifty observations of log linear data. The equation used is

$$Y = aX^{b} + error \tag{4-1}$$

where as in Chapter 3,

a is the first unit cost and is taken to be 1000,

X, is the unit number, and ranges from 1 to 50,

b is the learning rate. and is taken to be -0.3219 (80% curve), and

the error is uniformly distributed (one percent above and one percent below) about the expected value of Y for each unit X.

Once the data is generated, and the initial individual models are constructed, EXCEL's Solver Function is used to find the parameter values that minimize the Sum of Squared Errors (SSE). Each error is the residual computed by calculating the difference

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forecasted values can then be compared to the hold out set (the remaining observations from the given data set) to determine the fidelity of the fitted model.

between the fitted values and the actual values[2]. We deem the model with the smallest SSE to be the one which provides the best fit to the data.

The models and their respective equations are given in Table B-1 in Appendix B. Also included in this table are the parameter estimates which result from minimizing SSE. Note that asterisked parameters are the parameters which are adjustable during the optimization process. This table also includes the minimized values obtained for SSE.

The results are summarized below in Table 4-1 (This table is the same as Table B-2 in Appendix B.). This table is broken into two groups of model data. The first group summarizes the learning-curve models and the second group summarizes the ARIMA models. Note, also, that this table displays the rank of each model's SSE across the two groups. On the basis of SSE alone, the best overall model for curve fitting seems to be the basic Log-Linear Learning Curve model which comes in with a low SSE total of 763.7; this seems like an intuitive result since this is the same model which was used to produce the simulated data in the first place. The best *ARIMA model* for fitting this curve seems to be the AR(4) Model which comes in with an SSE of 1438.8. From this table, we also see that, based once again on the magnitude of SSE, the worst models for fitting seem to be the S-Curve, Pegel, and MA(1) models.

The SSE of the MA(1) and Pegel models turn out to be one and two orders of magnitude larger than the average SSE, which suggests they might be inferior to the other models. The poor performance of these models in *fitting* the Log-Linear Data base lead us to believe that they may also do poorly at *forecasting* this type of data. The Pegel model

[2] In this thesis, the term 'fitted value' refers to the per-unit value produced by the model which is fitted to the simulated data. The term 'actual value' refers to the (simulated) data to which the model is being fit.

is similar to the Forsythe model in that it contains a minimum cost term. Since the Forsythe model performed so much better than the Pegel in this fit test, we chose to keep it and to discard the Pegel model from futher consideration in this chapter. The results also confirm our suspicions from Chapter 3 regarding the MA(1) model; we recall that the MA(1) model did not appear to be well suited to modeling learning curves since rather than showing an extended downward trend similar to learning, the curve had reached its mean within just two time periods. Retaining the S-Curve for further study is questionable also since it not only performed poorly in the fit test, but also suffers from high complexity. For the reasons cited above, we drop these three models (MA(1), Pegel, and S-Curve) from further consideration in this thesis.

Table 4-1 Summary of Log-Linear Fitting

Model	SSE	Rank
Log-Linear	763.7	1
Forsythe	2256.4	4
Stanford-B	11576.1	11
Pegel	242678.9	13
S-Curve	43241.7	12
MA(1)	2665105.0	14
AR(1)	9685.7	10
AR(2)	4851.1	7
AR(3)	1756.4	3
AR(4)	1438.8	2
ARMA(1,1)	5856.5	8
ARMA(1,2)	6274.5	9
ARMA(2,1)	3714.9	5
ARMA(2,2)	4468.4	6

Plots which include the original data as well as the fitted curve for each model. are contained in Appendix B.

4-3. Fitting Historical F-102 Data

The data set for this section[3] was obtained from the document entitled Cost Functions for Airframe Production Programs (Womer, 1982) which draws its data from the F-102 Program Cost History (1965). This report says very little about the data. It tells us that the F102 program was comprised of 1000 aircraft which were constructed during the years 1953 through 1958. Further, it tells us that of these 1000 aircraft, 889 are F102A interceptors and 111 are TF102 trainers. Unfortunately, we are not told which aircraft are which within the data base, and can therefore not adjust for any irregularities this might produce in the data.

The variable of primary interest in the data base is the direct labor hours for each airframe; this is the column in Table E-1 (Appendix E) entitled TOTHRS. A portion of this table is shown in Table 4-2 for the reader's convenience.

Table 4-2 A Sample of the Historical F-102 Data Base

F-102 Data Base						
OBS	**PLN**	**DelaySeq**	**TOTHRS**	**Lot**	**Contract#**	**DM**
1	1	1	402475	1	5942	1
2	2	2	375849	1	5942	3
3	3	3	278963	2	5942	7
4	4	4	271223	2	5942	7
5	5	5	262498	2	5942	8
6	6	6	258078	2	5942	9
7	7	7	243726	2	5942	10
8	8	8	232766	2	5942	10
9	9	9	220833	2	5942	11
10	10	10	218827	2	5942	12
11	11	11	322447	3	5942	15
12	12	12	306736	3	5942	16
13	13	13	290470	3	5942	17
14	14	14	282951	3	5942	18
15	15	15	233125	4	5942	21

[3] The full data set is given in Appendix E . See the Epilogue at the end of this thesis for a short description of the search for this data.

The first question which arises is, "Against *what* should TOTHRS be plotted?" This is an important question because we note that there are two columns in the data base with headings that suggest they would be good candidates for the abscissa or unit number. The first possible column is labeled OBS; the second possible column is labeled PLN. Since we have no written description of the data in these columns, we make the following assumptions. We assume OBS is an indication of the relative position of an aircraft with respect to the end of the manufacturing process. For example, the first aircraft off the assembly line would be OBS 1, the second would be OBS 2, etc. We assume that the OBS number has nothing whatsoever to do with the position, in the manufacturing line, in which the plane *started.* We next assume PLN is an indication of the relative position of an aircraft with respect to the start of the manufacturing process. For example, the first aircraft started on the manufacturing line would be PLN 1, the second would be PLN 2, etc. We assume that the PLN number has nothing to do with the position in which the aircraft *finishes* the manufacturing process. For example, PLN 1 might, actually, not be completed until after PLN 2. So we could have a situation where PLN 1 could be the same aircraft as OBS 2! How do we decide which column of data to use?

To help us decide, we use EXCEL to sort the data first by OBS number and then by PLN number. We generate plots for each of these reordered data sets and present these in Figures 4-1 and 4-2. The most obvious problem seen in the figures is the large jump and then decline in the TOTHRS data at about PLN (and OBS) 11 - 15. Since it occurs in the both the plots of TOTHRS vs PLN and TOTHRS vs OBS, we can not choose which column to use based on this characteristic. Looking again, we also see

some small perturbations (spikes) in the dataplots. From these we choose to use TOTHRS vs PLN since the perturbations are much smaller in this reordered series.

Figure 4-1 Plot of Data -- TOTHRS vs OBS

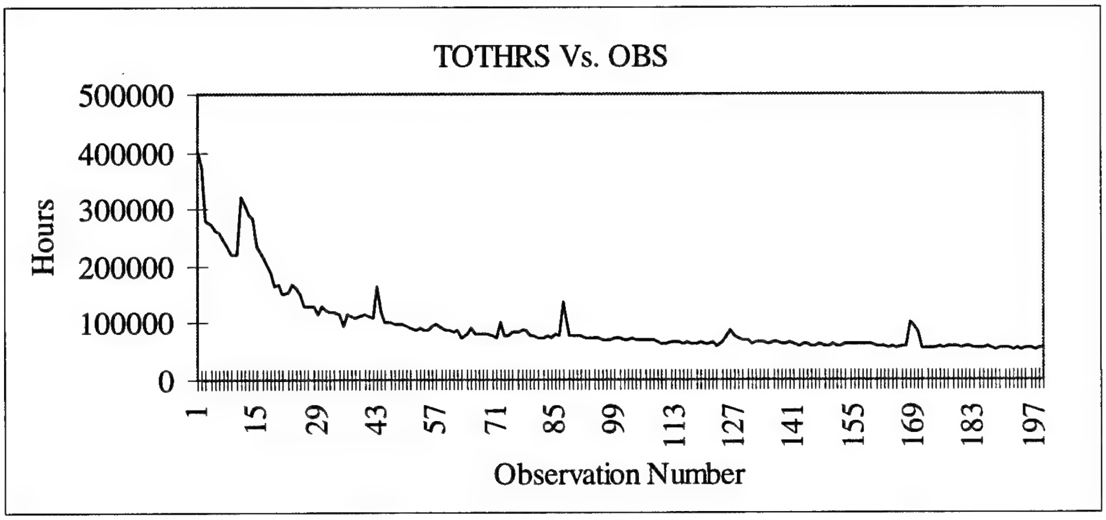
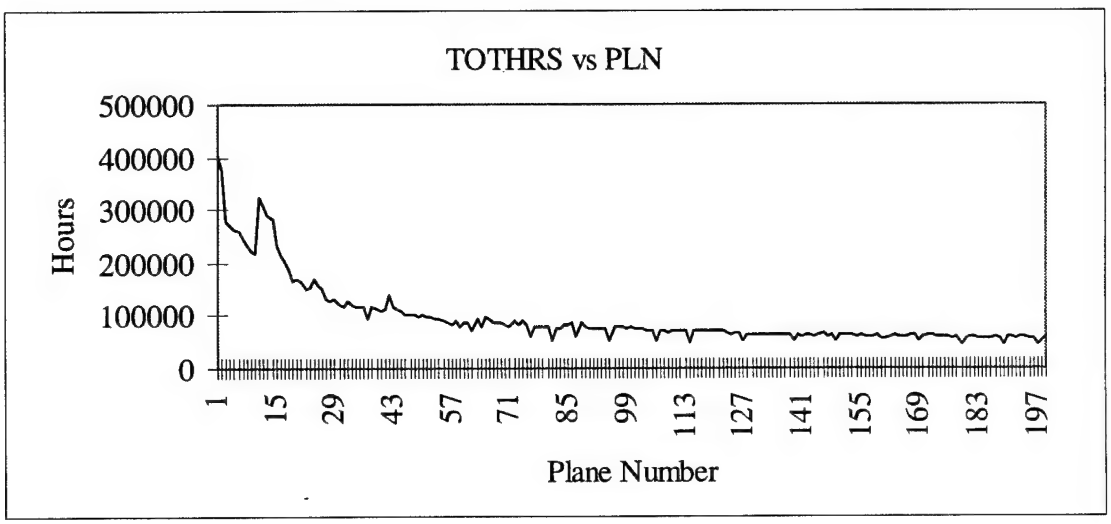


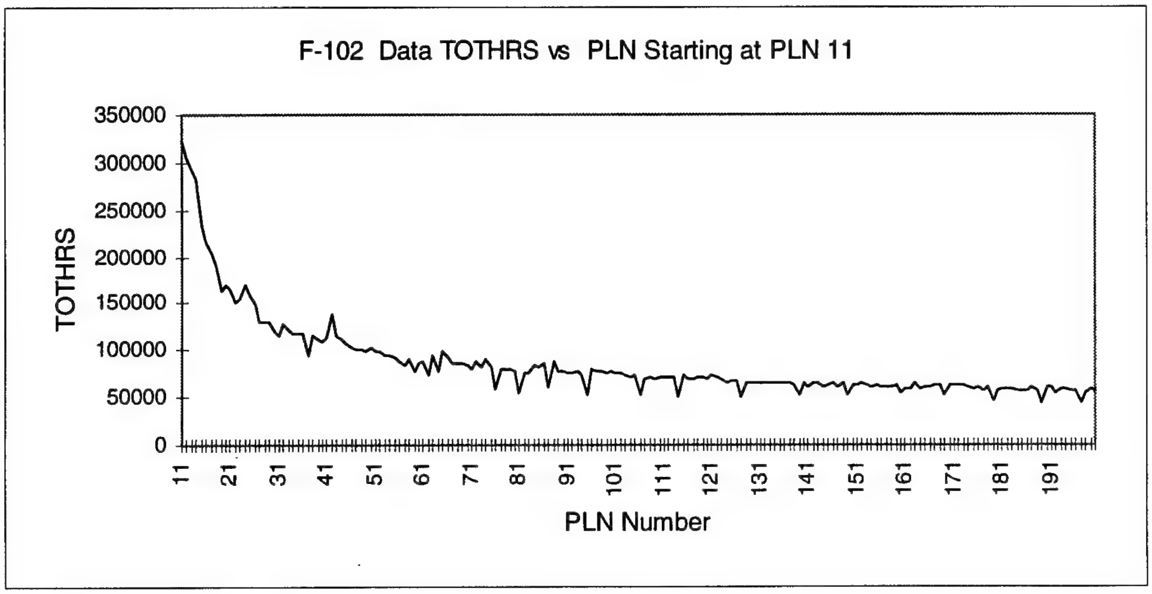
Figure 4-2 Plot of Data -- TOTHRS vs PLN



Adjusting for the 'jump' in the data which appears in both figures is just a little difficult since we have no documentation and can't be sure of the cause for this jump; it could be a major modification in aircraft design which required new learning. We could either ignore the jump or we could make some kind of adjustment for it so the models

might deal better with it. We seem to have two options for adjustment: we could either start at PLN number one and skip[4] Lot 3 (see column 5 in Table 4-2) Appendix D or column or we could just skip the first 10 observations (PLNs) and start the data at PLN 11. We arbitrarily decide to use the data starting at PLN 11. The resulting plot is shown in Figure 4-3.

Figure 4-3 F-102 Data (PLN vs TOTHRS) Starting at PLN 11



Once the data is adjusted, as described above, we are ready to start exercising the models. First thing we do is use EXCEL the same way we did in Section 2, to see how well each of the potential models can fit the F-102 data. As before, EXCEL's Solver Function is used to find the model parameters that minimize the Sum of Squared Errors (SSE). The models and their respective equations are given in Appendix C in Table C-1. Also included in this table are the parameter estimates that result from the use of the solver. Asterisked parameters are the parameters which are adjustable during the

[4] Lot 3 contains all four of the points which make up the 'jump.'

optimization process. The table also includes the minimum values for SSE. We again use SSE as the measure of performance to compare the models against one another.

The results of using the models to fit the F-102 data are summarized in Table 4-3 (This table is the same as Table C-2 in Appendix C.). Like Table 4-1, Table 4-3 is broken into two groups of model data. The first group summarizes the learning-curve models and the second group summarizes the ARIMA models. Note, also, that the table contains the ranks for each model with respect to SSE across the two groups. Plots which include the original data as well as the fitted curve for each model. are contained in Appendix C.

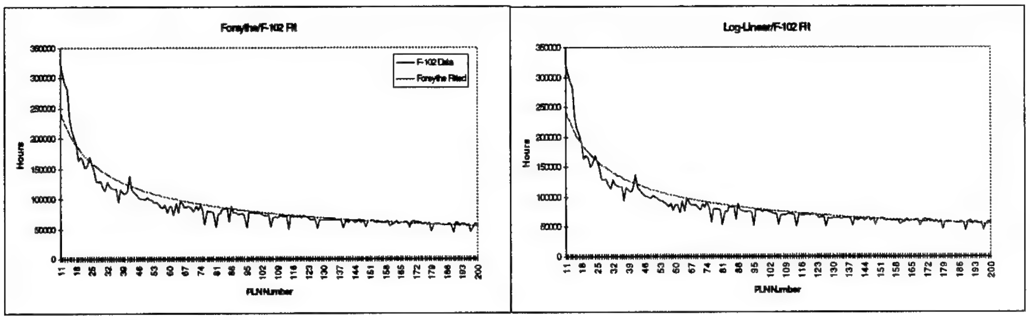
Table 4-3 Summary of F-102 Curve Fitting Investigation

Model	SSE	Rank
Log-Linear	5.503E+10	10
Forsythe	5.503E+10	10
Stanford-B	4.540E+10	9
AR(1)	2.722E+10	8
AR(2)	4.930E+09	2
AR(3)	2.637E+10	7
AR(4)	4.877E+09	1
ARMA(1,1)	2.089E+10	6
ARMA(1,2)	2.079E+10	4
ARMA(2,1)	2.087E+10	5
ARMA(2,2)	2.079E+10	3

The best over-all model for curve-fitting appears to be the AR(4) model which comes in with a low SSE total of 4.877e9. The best learning curve model for fitting this curve is the Stanford-B Model which comes in with an SSE of 4.54e10. Note that the SSE values, and hence the ranks, for the Standard Log-Linear and the Forsythe models are the same. This occurs because in this case, the solver estimated the parameter C_{min} to be zero in the Forsythe model ($Y = aX^b + C_{min}$); when this occurs, the Forsythe model is the same

as the Standard Log-Linear model ($Y = aX^b$). For purposes of comparison, plots of both fitted models as well as the F-102 data are shown in Figure 4-4. Note that although the plots below show the data and fitted model up through the 200th observation, the fit was actually performed using the observations up through 500.

Figure 4-4 Forsythe Model vs. Log-Linear Model in F-102 Fit



What have we done here? All we've done is use our potential models to see which can most closely follow the input data. In Section 4-2, we checked the models against a simulated log-linear data set. In Section 4-3, we checked the models against the F-102 data set. These experiments are good for giving us an idea which models might give a reasonable *fit* to this type of data but the only way to really find out which ones work and which ones don't work in forecasting, is to actually forecast with them!!

4-4. Fit/Forecasting Historical F-102 Data

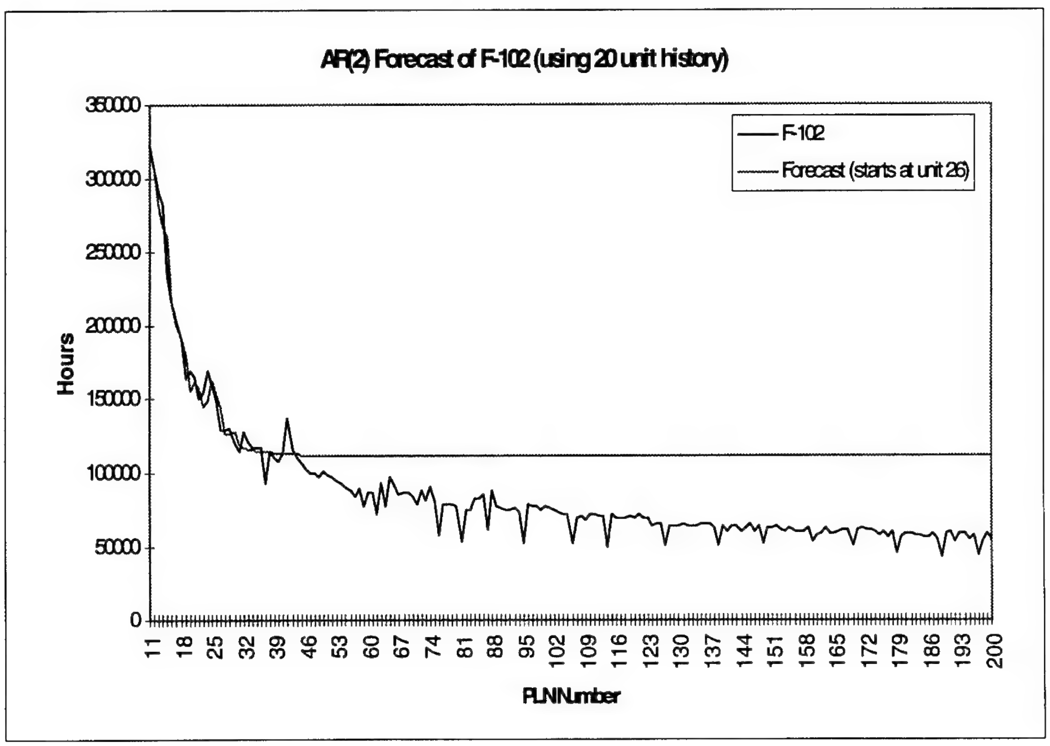
In this section, we use the F-102 data to test the ability of the potential models to forecast future build times. To accomplish this we use each of the models to fit the first twenty observations in the data set[5]. This is done in a manner similar to that used in Sections 4-2 and 4-3; we use EXCEL's solver to minimize the SSE by adjusting the

[5] Recall that the data set starts at PLN Number 11; the first twenty observations, therefore, consist of PLN Number 11 through PLN Number 30. The forecast starts with PLN Number 31.

parameter values. The output of the solver includes not only the minimized SSE, but the parameter estimates for each of the models. We use these parameters to build the forecast models. Once the forecast models are built, we use each of them to forecast out to unit 500. Then two more SSE's are calculated based on the hold out set of 480 and the full series of 500, as measures of each model's forecasting fidelity.

An example plot which includes the original data as well as the fitted AR(2) model (best forecast) is shown in Figure 4-5. Although the AR(2) provided the best forecast, we can see that the fit still isn't very good. The plots for the remainder of the models' forecasts are contained in Figures D-1 through D-11 in Appendix D.

Figure 4-5 AR(2) Forecast of F-102 Data



The rank results of the forecasting are summarized in Table 4-4 (This table is the same as Table D-2.).

Table 4-4 Summary of the F-102 Fit/Forecast Investigation

Model	SSE (1st 20)	SSE (last 480)	Rank (1st 20)	Rank (last 480)
Log-Linear	2.965E+09	6.175E+11	10	4
Forsythe	2.965E+09	6.175E+11	10	4
Stanford-B	2.814E+09	1.278E+10	9	2
AR(1)	2.452E+09	1.073E+12	8	6
AR(2)	2.350E+09	7.064E+09	6	1
AR(3)	2.257E+09	2.158E+12	5	9
AR(4)	1.707E+09	3.892E+10	4	3
ARMA(1,1)	2.414E+09	1.034E+12	7	5
ARMA(1,2)	1.153E+09	1.293E+12	3	7
ARMA(2,1)	8.848E+08	3.319E+12	2	10
ARMA(2,2)	8.793E+08	1.457E+12	1	8

If we examine the table, we see that it not only includes the SSE for the first 20 observations (the 'fitting' set) and for the last 480 observations (the 'hold-out' set), it includes ranking data which gives first place to the model with the lowest SSE. We can think of the SSE for the 'First 20' as a measure the goodness of the model's fit and the SSE for the 'Last 480' as a measure of the goodness of the model's forecast. Note that the ARMA(2,2) has the lowest SSE for the first twenty observations but ended up with only the eighth lowest SSE for the last 480 observations. This indicates that although this model provided the closest fit on the first twenty, it did not provide the best forecast for the entire sequence. The best forecast is provided by AR(2) which came in as the sixth best fit for the first 20. A model which provides the best forecast doesn't necessarily provide the best fit. Once again, the SSE values, as well as the ranks for the Standard Log-Linear and Forsythe models are the same because C_{min} in the Forsythe model was estimated to be zero by EXCEL's solver.

4-5. Fit/Forecasting Notional C-17 Data

In this section we use notional C-17 data to further test the ability of the models to fit/forecast future aircraft build times. Since the actual C-17 data is proprietary, we rescaled it so it would not show the actual production capability of the contractor. The data we use in this section, therefore, is not actual C-17 data but notional data.

Since the number of hours seems to increase between units 1 and 3.5, we start our test data set at observation 3.5. We are not sure why the hours increase within this range, but we can speculate that perhaps the early production units were used for some kind of testing which resulted in their completion taking longer than it normally might have. In order to give the models a slightly less confusing data set on which to work, we left out these observations and started, simply, at observation 3.5.

To test the ability of our potential models to forecast we proceed in a manner similar to that used in the previous section. We use each of the models to fit the first fifteen observations (observations 3.5 through 18) in the data set[6]. We then use EXCEL's solver to minimize the SSE by adjusting the parameter values. Once again, the output of the solver includes not only the minimized SSE, but the parameter estimates for each of the models. We use these parameters to build the forecast models. Once the forecast models are built, we use each of them to forecast build times for units 19 through 23. Finally, as before, two more SSE's are calculated based on the 'fitting set' of 15 and the 'hold-out set' of 8; these are utilized as measures of each model's forecasting fidelity. The results of the forecasting are summarized in Table 4-5 (This table is the same as

[6] Recall that the data set starts at PLN Number 3.5; the first fifteen observations, therefore, consist of Observation Number 3.5 through Observation Number 18. The forecast starts with Observation Number 19.

Table F-2 in Appendix F.). Plots which include the original data as well as the fitted curve for each model. are contained in Appendix F.

Table 4-5 Summary of the Notional C-17 Fit/Forecast Investigation

Model	SSE (1st 15)	SSE (all 23)	SSE (last 8)	Rank (1st 15)	Rank (all 23)	Rank (8)
Log-Linear	870.7	1040.1	169.4	6	6	3
Forsythe	291.1	386.5	95.4	2	2	2
Stanford-B	359.2	364.7	5.5	5	1	1
AR(1)	1266.9	1727.7	460.8	7	8	10
AR(2)	1340.4	1753.9	413.6	9	9	9
AR(3)	1405.5	1754.0	348.4	10	10	8
AR(4)	1302.1	1503.5	201.4	8	7	4
ARMA(1,1)	89.9	398.1	308.2	1	3	7
ARMA(1,2)	3699.0	10270.0	6571.0	11	11	11
ARMA(2,1)	329.9	572.3	242.3	4	5	6
ARMA(2,2)	302.9	524.1	221.2	3	4	5

Note that the table not only has the actual SSE's for the fit set, the hold-out set, and the entire set, it has corresponding columns of ranks for each model. From table 4-5, we see that based on the SSE values of the hold-out set, the first and second best forecasts were turned in by the Stanford-B and the Forsythe Models with SSEs of 5.5 and 95.4 respectively.

4-6. Summary

The results of this chapter's investigations is summarized in Table 4-6.

Table 4-6 Investigation Summary -- Ranks Based on SSE's

Investigations: → Models/Ranks: ↓	(Section 4-2) Fitting Log-Linear Data	(Section 4-3) Fitting F-102 Data	(Section 4-3) Fit/Forecast F-102 Data	(Section 4-4) Fit/Forecast Notional C-17 Data
First Best	Log-Linear	AR(4)	AR(2)	Stanford-B
Second Best	AR(4)	AR(2)	Stanford-B	Forsythe
Third Best	AR(3)	ARMA(2,2)	AR(4)	Log-Linear
Fourth Best	Forsythe	ARMA(1,2)	Log/Lin and Forsythe	AR(4)

In Section 4-2, we investigated the ability of the candidate models to fit a simulated log-linear data set. The standard Log-Linear Learning Curve model provided the best fit to the data; this makes perfect sense since it was the standard Log-Linear Learning curve which produced the data in the first place! The second and third best fits were provided by the AR(4) and AR(3) models respectively; their SSE statistics were very nearly the same (1438.8 vs 1756.4) so if we had to chose a model to fit log-linear data based on this investigation, the concept of parsimony would probably have us choose AR(3).

In section 4-3, we investigated the ability of the candidate models to fit historical F-102 data. In the fit test, the models which provided the first and second best fits to the 500 observation data set were the AR(4) and AR(2) models respectively. Their SSE statistics were very close (4.977E+09 vs 4.930E+09), so if we had to choose a model to fit the F-102 the concept of parsimony would have us choose the AR(2) model.

Also in section 4-3, we developed forecast models for the F-102 data by using hold-out sets; we used the first 20 observations to fit the forecast model and went on to test it against the 480 observation hold-out data set. Based on SSE values alone, the first and second best forecast models appear to be the AR(2) and Stanford-B models respectively. The SSE for the AR(2) was only about half as large as the SSE for the Stanford-B. In third place was the AR(4) model with an SSE about twice the value of the Stanford-B value. One interesting thing to note is that the model which provided the best fit to the first 20 observations did not provide the best forecast. The ARMA(2,2) model provided the best fit to the first 20 observations but came in with the 8th best forecast.

In section 4-4, we developed forecast models for the notional C-17 data by using hold-out sets. The fit set consisted of the first 15 observations while the hold out set consisted of the last 8 observations. The first, second, and third best forecasts seem to be given by the Stanford-B, the Forsythe, and the Log-Linear Learning Curve models respectively. Once again, the models which provide the best forecasts did not necessarily provide the best fits. The Stanford-B, the Forsythe, and the Log-Linear Learning Curve models provided the 5th, 2nd, and 6th best fits, respectively.

The results are somewhat inconclusive in that the study does not identify any one model as always being the best fitting or forecasting tool for all data sets. It does, however, provide an important general observation that ARMA models are a promising alternative to the standard log-linear learning curve approach which is widely in use today; they are comparatively simple to use, intuitive in nature and seem to provide a good forecast based on a small amount of data.

5. Simulating Future Performance

5-1. Problem Statement

The purpose of this thesis up to this point has been to investigate existing learning curve models and to examine alternative models which, based on initial production data, might better predict the amount of time future aircraft builds will take. One of the things we have addressed only briefly, if at all, has been the uncertainty in fit of the models under study. In the C-17 Factory Simulation Model, predictions of future performance were made by treating the fitted regression as if it provided a *perfect* forecast of the mean time required to complete a task in the future. Once this relationship was determined, random errors were generated about the fitted curve to simulate how actual performance might vary in the future. In this case, the simulation team felt they could adequately predict future performance by *ignoring* the uncertainty of the fitted regression.

The goal of this chapter is to explore various methods for addressing the following questions. Given a learning curve model, how can we best account for the uncertainty in fit? Should we just ignore it as has been done in previous aircraft manufacturing simulations? Is this a sound procedure? Alternately, should we adjust the variance of future observations to account for uncertainty? Or should we sample from a distribution of parameter estimates in each replication? We explore this through the use of basic meta-modeling concepts applied to the learning curve model.

5-2. Simulation Based on a Fitted Linear Metamodel

What is a metamodel? The following definition, lifted from the dictionary (Meriam-Webster, 1977), gives a small amount of insight into the meaning of the meta (or met) prefix and starts us on our way to understanding what a metamodel is!

> **meta-** or **met-** *prefix* [from Latin, change, or Greek, among, with, after] **1a:** occuring later than or in successsion to : after **1b:** situated behind or beyond **1c:** later or more highly organized or specialized form of **2:** change : transformation **3:** more comprehensive : transcending <*meta*psychology> -- used with the name of a discipline to designate a new but related discipline designed to deal critically with the original one <*meta*mathematics>

To put it more succinctly, Russell R. Barton (Barton, 1992) says, "a metamodel is a model of a model."

Let's look at an example of a metamodel. We first use an equation, say the learning curve equation ($Y = aX^b$ + error), to generate a data base; we did this in Chapter 3 when we generated the first fifty observations of a specific learning curve. We next use the technique of Ordinary Least Squares (OLS regression), to 'fit' a predictive model to that data. What we end up with are parameter estimates defining a fitted model which characterizes the data base we have generated (a model of a model!). When we make a (regression) model of the simulation model, we speak of a (regression) *metamodel.* (Kleijnen, 1992)

In this section, we examine strategies for generating simulated values from a fitted linear model. We, in particular, want to simulate future performance; in other words, we want to extrapolate outside of the design region of the fitted model. The case in which we are interested, and with which we start, is the case where the independent variable, X, represents time, or some function of time. We assume we have observed the dependent variable, Y, for values of the independent variable ranging from X = 1 to X = n. What we wish to do is to repeatedly simulate

future sequences of the dependent variable, Y, for values of the independent variable ranging from $X = n+1$ to $X = n + k$ for some specified k.

For simplicity, let's assume that the relationship between a dependent variable, Y, and an independent variable, X, can be described via a simple linear model of the form

$$Y = \alpha + \beta X + \in, \tag{5-1}$$

where $\in \sim N(0, \sigma^2)$. If we **know** the values of the parameters α, β, and σ^2, we can say that

$$Y \sim N(\alpha + \beta X, \sigma^2) \tag{5-2a}$$

or, equivalently,

$$\frac{(Y - \alpha - \beta X)}{\sigma} \sim N(0, 1). \tag{5-2b}$$

Better yet, and still equivalently, we could denote this as

$$Y = \alpha + \beta X + \sigma[N(0,1)] \tag{5-2c}$$

Where the last term in (5-2c) indicates that we have added to the mean $(\alpha + \beta X)$ a standard deviation term which is normally distributed with a mean of zero and a standard deviation of one. We can use this to generate simulated values of Y for given values of X. If we wish to generate a value of the dependent variable for, say, $X = X_h$, we can use the following simple algorithm:

1. Generate $Z \sim N(0, 1)$.
2. Set $Y = \alpha + \beta X_h + \sigma Z$.

For our purposes we assume, for simplicity, that $X_h = h$. This allows the simulated sequences $(Y_{n+1}, Y_{n+2}, \ldots, Y_{n+k})$ to be generated in a relatively straightforward manner. So, when we know the values of the parameters α, β, and σ^2, there *is* no uncertainty in the fit since the mean and standard deviation are known.

The uncertainty problem arises when we *DON'T* know the values of these parameters. This leads us to the first of three methods of dealing with uncertainty in fit.

Method 1

If we do not know the values of the parameters α, β, and σ^2, which is most often the case, they must be estimated from sample data. Let's assume we observe the data points (X_1,Y_1), (X_2,Y_2), . . . , (X_n,Y_n); from these, we can estimate α, β, and σ^2 via ordinary least squares. The least squares estimates of these parameters turn out to be,

$$a = \overline{Y} - b\overline{X}, \tag{5-3}$$

$$b = \frac{\sum X_i Y_i - \frac{\sum X_i \sum Y_i}{n}}{\sum X_i^2 - \frac{(\sum X_i)^2}{n}}, \tag{5-4}$$

and,

$$\hat{\sigma}^2 = MSE = \frac{\sum (Y_i - a - bX_i)^2}{n-2} \tag{5-5}$$

(Neter, Wasserman, and Kutner: 1990, pgs 50, 145). We know from basic statistics that, in this case,

$$E(a) = \alpha, \tag{5-6a}$$

$$E(b) = \beta, \tag{5-6b}$$

and

$$E(MSE) = \sigma^2. \tag{5-6c}$$

Furthermore, in most statistics texts, the predicted value of Y at $X = X_h$ is generally denoted as $\hat{Y}_h = a + bX_h$. Under the assumptions we have made, we can say that

$$E(\hat{Y}_h) = E(a + bX_h). \tag{5-7}$$

and, since we know (as stated above in equations 5-6) that a, b, and MSE are unbiased estimators of α, β, and σ^2, respectively, we can go a step further to say

$$E(\hat{Y}_h) = E(\alpha + \beta X_h)\,. \tag{5-8}$$

All of this having been said, we might be tempted, for simplicity, to generate simulated values of Y at $X = X_h$, via the algorithm:

1. Generate $Z \sim N(0, 1)$.
2. Set $Y = \hat{Y}_h + (MSE)^{1/2} Z$ (5-9a)

or, equivalently,

$$Y = a + bX_h + (MSE)^{1/2} Z. \tag{5-9b}$$

This method yields a distribution of Y's which has a constant variance, as we would expect, and is very similar to the strategy implemented in the recent Factory Simulation Model of the C-17 assembly process. The factory simulation model is consistent with the model we postulate above in that it has the property that the variance of the simulated Y's is constant and *not* dependent on X_h. Unfortunately, because it is implicitly based on the assumption that

$$Y \sim N\left(\hat{Y}_h, MSE\right) \tag{5-10a}$$

or, equivalently,

$$\frac{(Y - \hat{Y}_h)}{(MSE)^{1/2}} \sim N(0,1)\,. \tag{5-10b}$$

we find that our postulated model cannot be strictly correct. We say this because according to Neter, Wasserman & Kutner (1990), for example, that

$$Var(\hat{Y}_h) = \sigma^2\left[\frac{1}{n}+\frac{(X_h-\overline{X})^2}{\sum(X_i-\overline{X})^2}\right] \tag{5-11}$$

and thus that the quantity

$$\frac{Y-\hat{Y}_h}{\sqrt{(MSE\left[1+\frac{1}{n}+\frac{(X_h-\overline{X})^2}{\sum(X_i-\overline{X})^2}\right]}} \tag{5-12}$$

(which is proportional to the quantity on the left hand side of equation 5-10b) does *not* have a N(0,1) distribution as stated above. Instead of an N(0,1) distribution, it has a Student's -t distribution with n-2 degrees of freedom. Based on this realization, we can propose another strategy which might be more accurate.

Method 2

The next possible strategy is given by the following algorithm.

1. Generate T ~ t(n-2).
2. Set $Y = \hat{Y}_h + \sqrt{MSE\left[1+\frac{1}{n}+\frac{(X_h-\overline{X})^2}{\sum(X_i-\overline{X})^2}\right]}T$ (5-13)

A potential weakness of this strategy is that the variance of the simulated Y's is not constant but, as can be seen from inspection of equation (5-13), depends on X_h. In fact, the variance of the simulated Y's *increases* as the distance between X_h and $\overline{X}$ increases. Although this strategy correctly accounts for the uncertainty within our estimates of the parameters α and β, it is somewhat undesirable since the behavior is not consistent with that which would be expected from the real system[1]. This would be especially unappealing in the

[1] In a real system, we would expect the variance to remain constant.

time-dependent case where we successively generate values of Y corresponding to values of X ranging from X = n+1 to n+k. Finally, we are led to a third method which attempts to deal with this shortcoming.

Method 3

The third method takes into consideration the fact that, since a and b are normally distributed, the standardized statistics (b-β)/s(b) and (a-α)/s(a), where s(a) and s(b) are estimates of the standard deviation of a and b respectively, are distributed as t with n-2 degrees of freedom. Since the estimates of the standard deviations, s(a) and s(b), are given by

$$\sqrt{MSE\left[\frac{1}{n}+\frac{\overline{X^2}}{\sum(X_i-\overline{X})^2}\right]} \tag{5-14}$$

(Neter, Wasserman & Kutner: 1990, pg 71), the standardized statistics (b-β)/s(b) and (a-α)/s(a) become

$$\frac{a-\alpha}{\sqrt{MSE\left[\frac{1}{n}+\frac{\overline{X^2}}{\sum(X_i-\overline{X})^2}\right]}} \tag{5-15a}$$

$$\frac{b-\beta}{\sqrt{\frac{MSE}{\sum(X\ -\overline{X})^2}}} \tag{5-15b}$$

and are distributed as t(n-2).

This in mind, a possible strategy for generating simulated values of Y for X=X_h is as follows:

1. Generate T_1 and T_2 ~ t(n-2)

2. Set A = $a+\sqrt{MSE\left[\frac{1}{n}+\frac{\overline{X^2}}{\sum(X_i-\overline{X})^2}\right]}*T_1$ (5-16a)

3. Set B = $b+\sqrt{\frac{MSE}{\sum(X_i-\overline{X})^2}}*T_2$ (5-16b)

4. Generate Z ~ N(0,1)

5. Set Y = A + BX_h + $(MSE)^{1/2}$Z. (5-16c)

If a sequence of values starting at X = n+1 is desired, one could repeat steps 3 and 4 for each successive sequence of variables. Although the resulting values of Y will, in the aggregate, have a variance that increases as the distance between X_h and $\overline{X}$ increases, if we apply steps 3 and 4 iteratively for given values of A and B, the individual sequences generated will behave like observations from a linear model with a constant variance.

With all of this theory behind us, we next demonstrate each of the three methods using, first, a linear model, and second, a log-linear model.

5-3. Linear Case Studies

To simplify the development process, we start with modeling and simulating a simple linear relationship. Once the methods outlined above are demonstrated using this example, we go on, in the next section, to apply them to the Log-Linear Learning Curve equation.

Generating 20 Simulated Observations: Here we assume we know the values of the parameters α (intercept), β (slope), and σ^2 (fixed variance). Using EXCEL, we start out by simulating a set 20 observations calculated from the following equation.

$$Y = \alpha + \beta X + \sigma[N(0,1)] \quad (5\text{-}17)$$

We've added the σN(0,1) term to stochasticize the simulated observations!

Once the data is simulated, we use the method of ordinary least squares to fit a linear regression model to the 20 simulated observations; this, of course, yields estimated values for α, β, and σ^2 (a, b, and MSE, respectively). So, we've made a model of a model. These fitted values are used throughout the following three methods as the basis in simulating future values of the dependent variable.

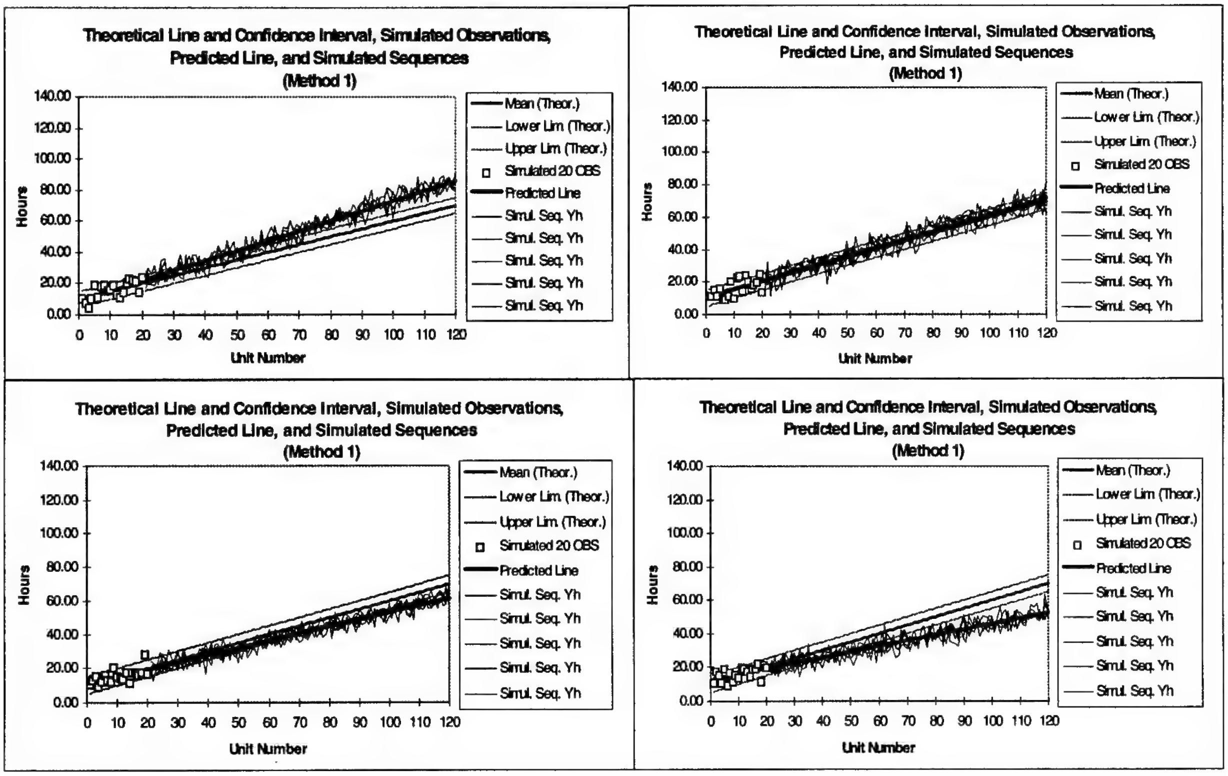
Method 1:

Once the theoretical line and confidence interval are plotted, we simulate and plot five future sequences of values Y_h for $X = X_h$ for n = 21 through n = 120 on the same set of axes. To calculate these sequences, we use equation (5-9b). This done, we repeat starting with three more sets of 20 initial observations and recalculate the plot three times to show the random nature of the simulated data and the uncertainty in fit of the predicted line. These plots are shown below, in Figure 5-1 for illustration.

Note, in the plots, how the random nature of the simulated data has a significant effect on the the fitted regression line. The fitted line does not always accurately model the true relationship; a small error in the estimate of slope can seriously skew the fit away from the simulated data in the long run. As might possibly have been uttered before, perhaps we *shouldn't* treat the fitted regression as if it provided a *perfect* forecast of the expected value in the future.

We also note that the variance of each individual simulated sequence is, by construction, constant over time. This is as we would expect from a real life system.

Figure 5-1 Method 1 for Equation of Line -- Four Simulations



We must keep in mind that the calculations done to generate the preceding simulated sequences were not entirely correct since they were built upon the assumption, as given in equation (5-10b), that

$$\frac{(Y-\hat{Y}_h)}{(MSE)^{1/2}} \sim \mathrm{N}(0,1)$$

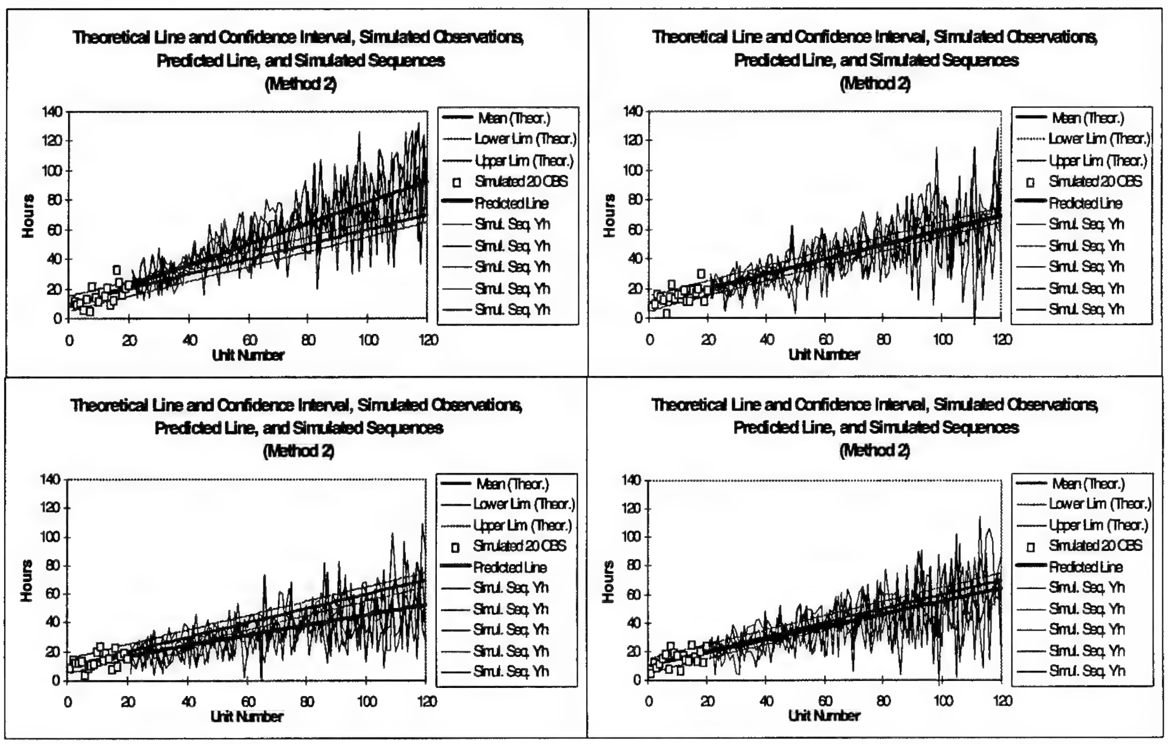
when in actuality, it is proportional to the student's-t distribution. Method 2 will work the student's-t distribution into the calculation of the Y_h's.

<u>Method 2</u>

In a manner analogous to the techniques used in Method 1, we again generate 20 simulated observations, use them to fit a curve, and then predict five series of Y_h for X = 21 to

X = 120. To calculate these new sequences, we use equation (5-13). The only difference here is, as outlined in Section 2, Y is estimated as $(Yhat)_h$ plus a standard deviation times a student's t distribution. The standard deviation term is dependent on X_h and grows in magnitude as we go forward in time; see Figure 5-2. Once again we can observe the uncertainty in fit of the regression line and the five simulated sequences; their slopes vary from plot to plot. This could be the case in subsequent experiments since the simulated sequences are built upon the fitted regression line.

Figure 5-2 Method 2 for Equation of Line -- Four Simulations

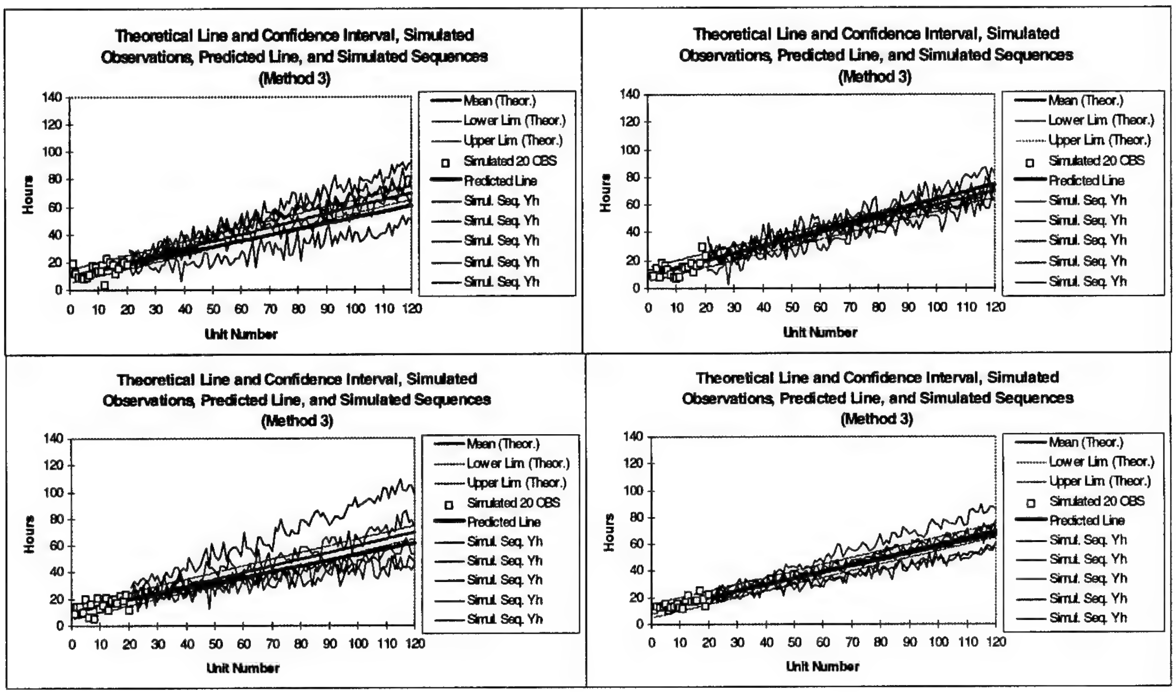


The variance, unfortunately, is not constant as we expect to find in the real life system. To compensate for this disparity between the simulated sequences and what we expect in reality, we go on to Method 3 which provides us with simulated sequences which each appear to have relatively constant variance through the range of X_h's.

Method 3

Following the same technique used for Methods 1 and 2, above, we can, once again, generate 20 simulated observations, use OLS regression to fit a line, and then predict five series of Y_h for X = 21 to X=120 for comparison to the theoretical line. To calculate these sequences, we use equations (5-16a and 5-16b). The resulting plots are shown in Figure 5-3.

Figure 5-3 Method 3 for Equation of Line -- Four Simulations



Note how *each* of the five simulated series of Y_h's, for Xh = 21 through Xh = 120, unlike the series produced in Methods 1 and 2 above, all go off in slightly different directions while at the same time maintaining a constant variance within each individual series. What this helps us to see is that we can take into account the uncertainty of the fit of the regression line by simulating not just one but multiple series of future values for Y_h. In this way, we build a confidence interval of sorts (made up of the plots of the individual sequences) which helps us to keep in

mind the fact that the regression line and its confidence intervals do not necessarily provide a perfect fit for what the actual data might be expected to do! In the C-17 Factory Simulation, little or no effort was made to account for this uncertainty.

In the next section (Section 4), we simulate the first twenty observations of the *learning curve* the same way we simulate them for the equation of the line above. As before, we also go on to fit this simulated data (building a model of a model) using a log transformation and OLS regression. The goal of the next section is to investigate the nature of the uncertainty in the Log-Linear Learning curve forecasts and to determine if we even *need* to account for it in the construction of our simulated sequences of Y_h's. Because of the rescaling produced by the log transformations, the discrepancy in fit which we see in the linear case may not be apparent in the case of the log-linear learning curve.

5-4. Log-Linear Learning Curve Case Studies

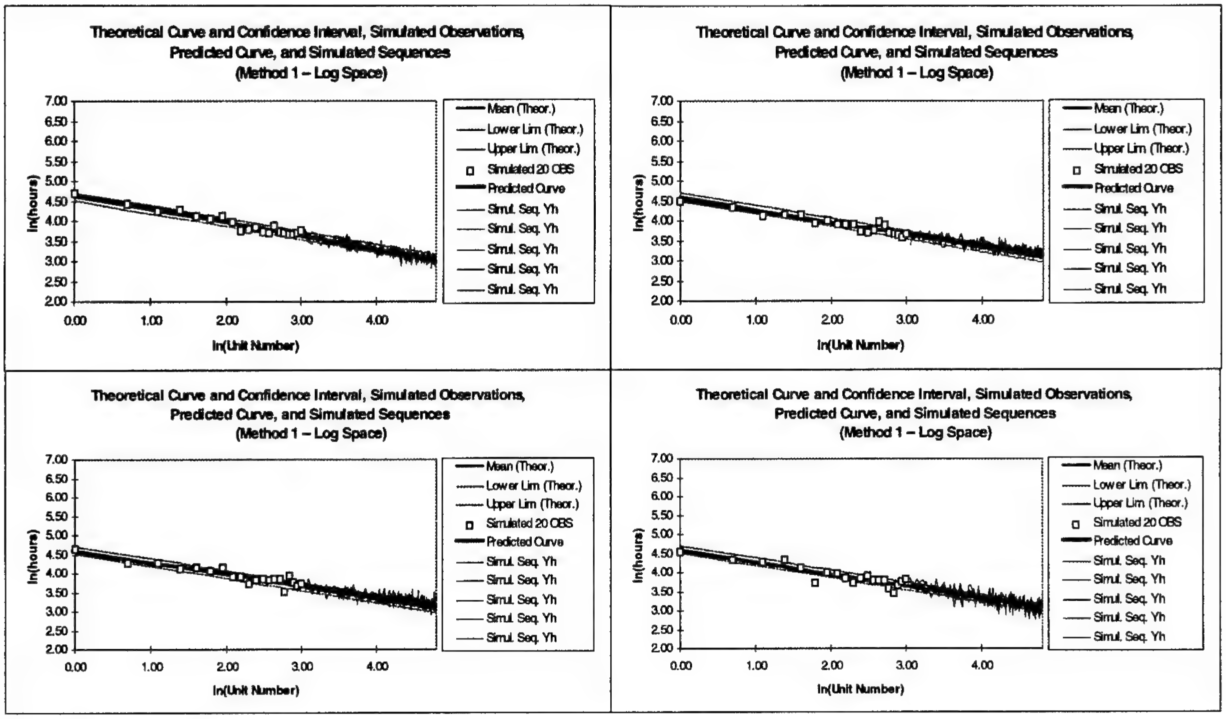
Although we discovered with the linear cases above that the uncertainty in fit could produce significant inaccuracy, we may *not* discover that the uncertainty for the log-linear case is worth accounting for. Since this is difficult to know without investigation, we shall now go on to see what effect Methods 1, 2, and 3 have on the associated plots for the learning curve.

<u>Method 1</u>

In a manner analogous to the techniques used in Method 1 (Section 5-3), we again generate 20 simulated observations; this time we use the log-linear learning curve equation $Y = aX^b + \text{error}$. We then fit a curve to these observations, and predict five series of Y_h for $X = 21$ to $X = 120$. To calculate these new sequences, we use equation (5-9b). Again, we recalculate the simulated observations, the regression curve, and the simulated sequences of Y_h's.

These are all shown in Figure 5-4. We note, from the four simulations in Figure 5-4, that the fit of the regression line varies almost indiscernably from plot to plot.

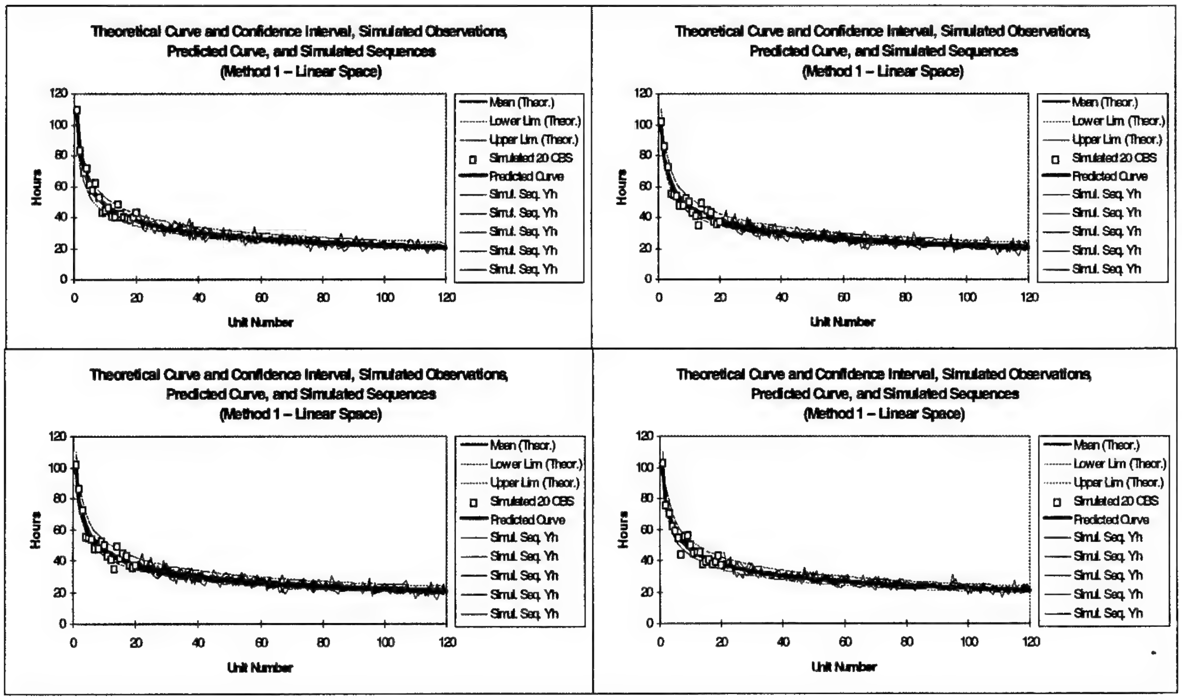
Figure 5-4 Method 1 for Log-Linear Equation *In Log Space* -- Four Simulations



In addition to the plots in log space shown in Figure 5-4, we also generated plots in linear space by plotting hours vs.unit number instead of plotting the natural log of hours versus the natural log of the unit number as we did for the plots in log space; the plots in linear space are shown in Figure 5-5. We can make the same observations about these plots that we made regarding those in Figure 5-4; the fit of the regression curve varies almost indiscernably from plot to plot. Since Method 1 is very similar to the method used in the C-17 FSM, we start to think, based on these plots, that ignoring the uncertainty in fit might not have the high price-tag which we originally expected. Despite these early indications that we might not need to account for

uncertainty (at least not in the case of the log-linear learning curve), we'll look at the effects that Methods 2 and 3 have on the fitted regression curves.

Figure 5-5 Method 1 for Log-Linear Equation *In Linear Space* -- Four Simulations



Method 2

Once again, we generate four sets of simulated observations, four fitted regression lines, and four sets of five sequences of simulated future observations (Y_h's). This time, as in Method 2, Section 5-3, we use equation (5-13) which corrects the incorrect assumption regarding the distribution of the error which is actually distributed as t(n-2) *not* N(0,1). We plot the resulting data both in log space and in linear space; these plots are shown in Figures 5-6 and 5-7 respectively. In Figure 5-6, we see the same increasing variance evident in Method 2, Section 5-3. Once again, however, the fitted regression curve seems not to vary noticiably from simulation to simulation.

Figure 5-6 Method 2 for Log-Linear Equation *In Log Space* -- Four Simulations

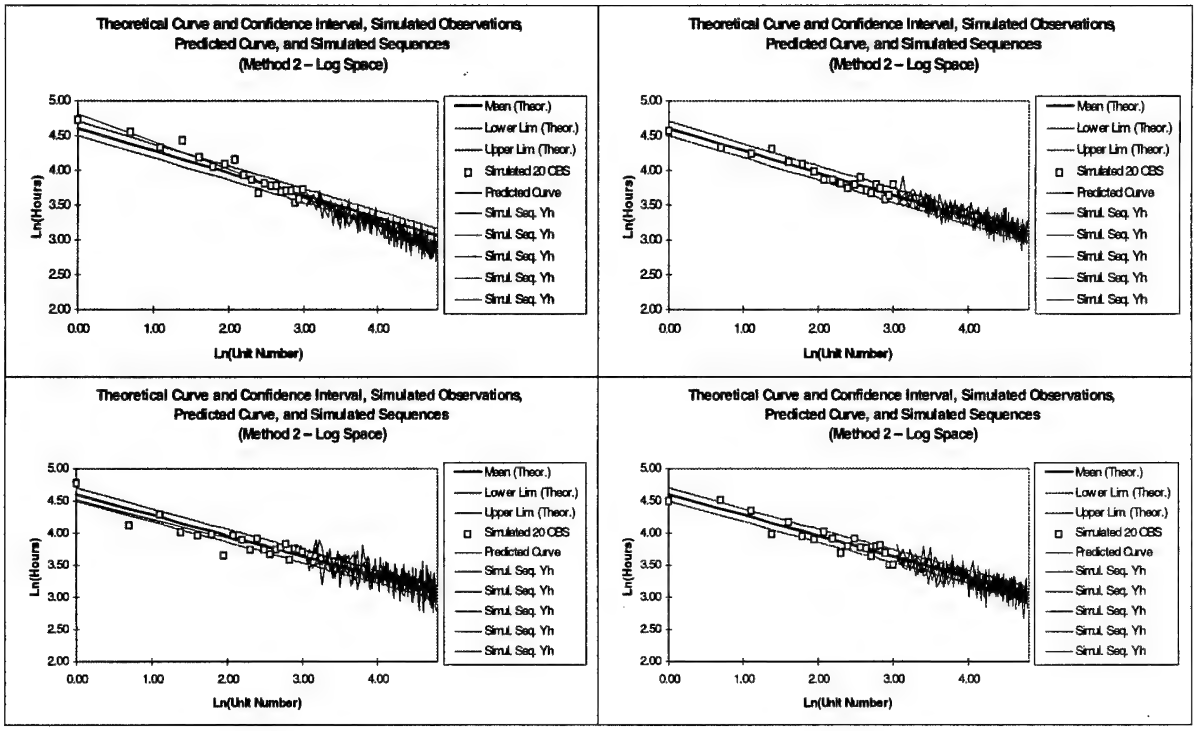
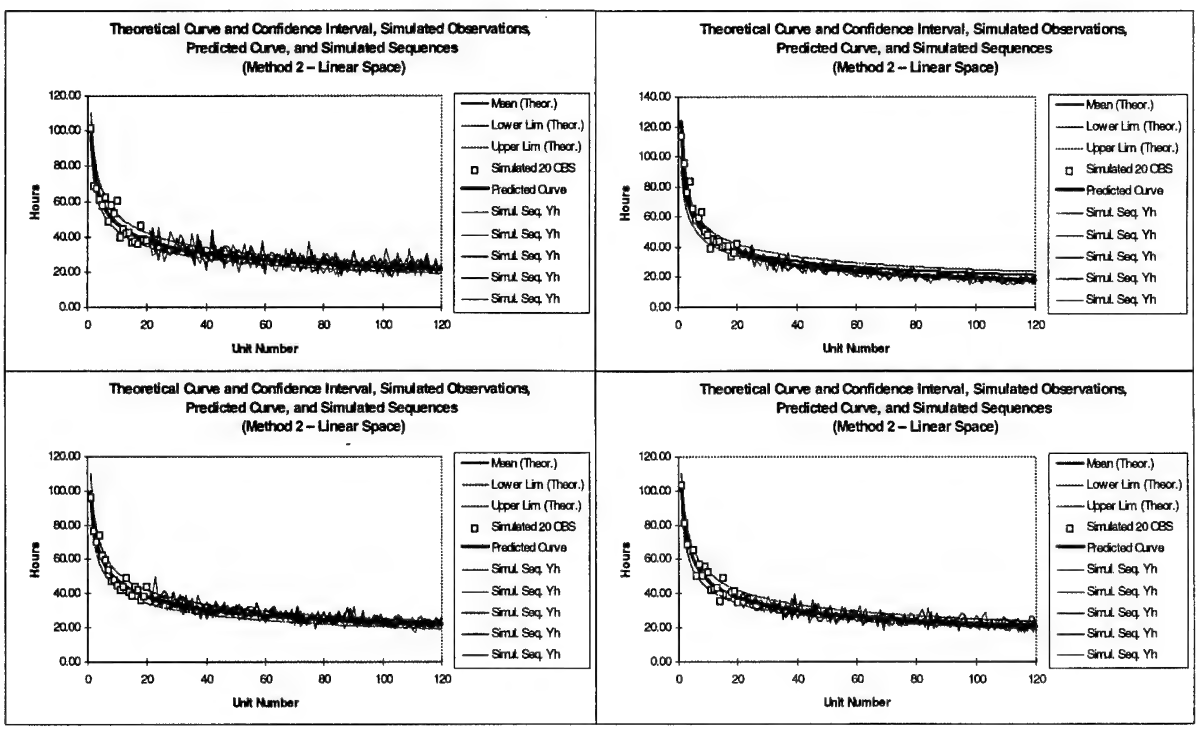


Figure 5-7 Method 2 for Log-Linear Equation *In Linear Space*-- Four Simulations



Method 3

One last time, we generate the 20 simulated observations, the fitted regressions, and the simulated sequences of future observations. This time when we generate the simulated sequences, we use equations (5-16a through 5-16c). The result is a series of simulated sequences each of which seem to have a constant variance. Once again, we've gone on to plot the resulting data in both log and linear space; these plots are shown below in Figures 5-8 and 5-9.

Figure 5-8 Method 3 for Log-Linear Equation *In Log Space* -- Four Simulations

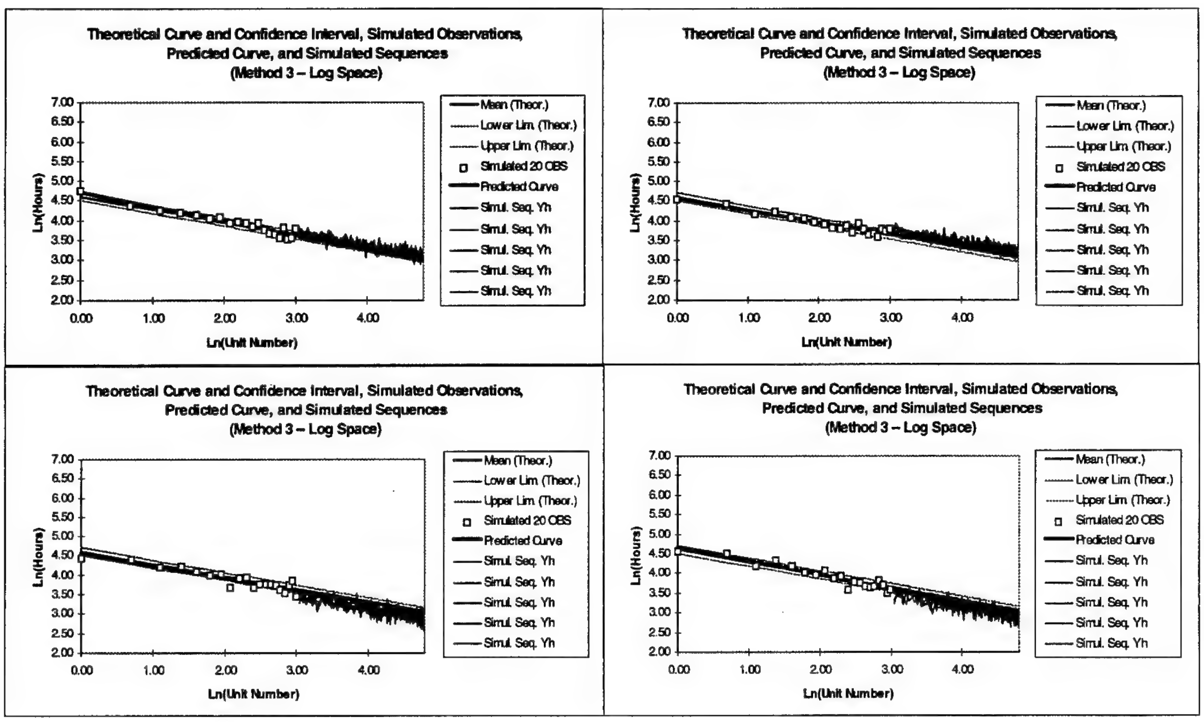
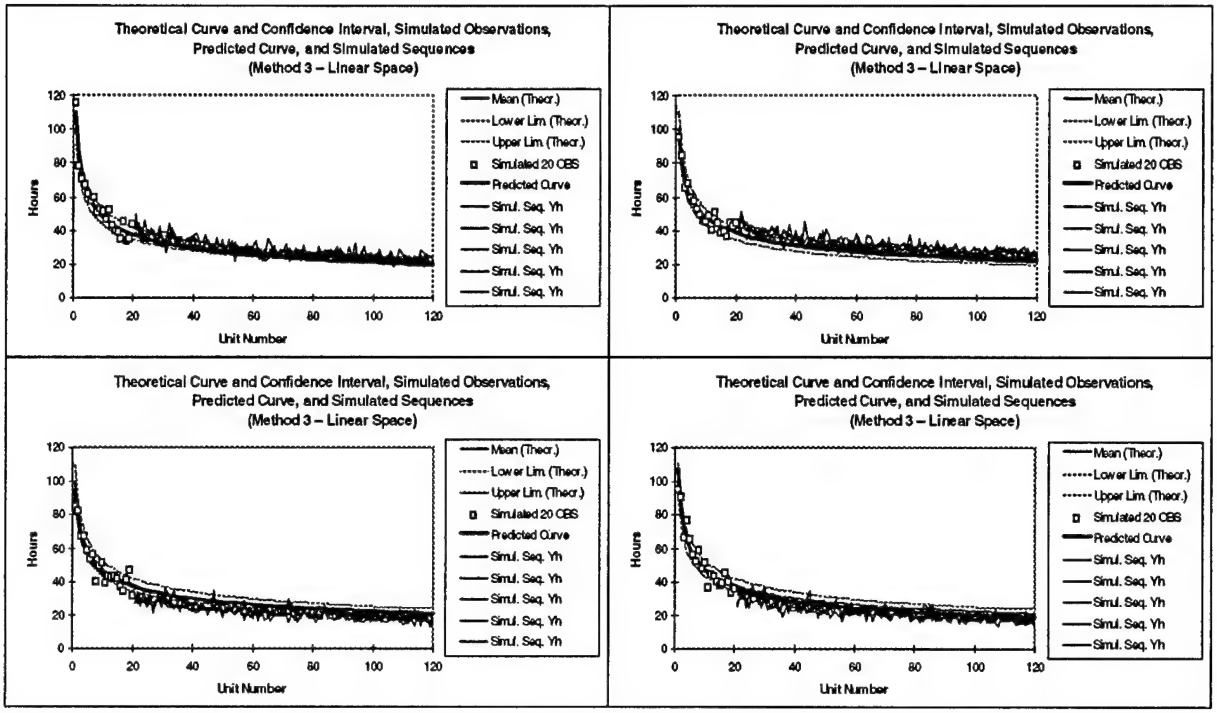


Figure 5-9 Method 3 for Log-Linear Equation *In Linear Space* -- Four Simulations



5-5. Conclusions

From the plots in Figures 5-4, through 5-9, we can say that perhaps the C-17 FSM was not seriously injured by the fact that during its development, uncertainty in fit was ignored. Method 1, which is similar to the methods used in the C-17 FSM, produced reasonable fits, not only in Log-Space, but in linear space as well. Method 2 and Method 3 seemed to make no appreciable amount of inprovement in the fits.

6. Conclusions

The results of this thesis are somewhat inconclusive since the study does not identify any one model as always being the best fitting or forecasting tool for all sets of data. It does, however, provide an important general indication that ARMA models, particularly the AR models, are a promising alternative to the standard log-linear learning curve approach which is widely in use today; they are comparatively simple to use, intuitive in nature and seem to provide a good forecast based on a small amount of data.

The investigation of metamodels and the question of accounting for the uncertainty in fit within the C-17 Factory Simulation Model yielded some encouraging, as well as potentially useful, results. In the case of the simple linear model, we found that the application of our proposed Methods 2, and especially, 3 did a good job of accounting for uncertainty within the fit of the regression line; the resulting simulated sequences seemed to cluster well around the theoretical confidence interval. In the case of the log-linear learning curve, however, the results were not as striking. In particular, we found that our Method 1, which is very similar to the methods used in the C-17 FSM, did a reasonable job of simulating sequences of (future) observations which, for the most part, fell within the theoretical prediction interval. Application of Methods 2 and 3 did not noticeably improve the fidelity of the simulated sequences. The bottom line, here, is that ignoring the uncertainty in fit (as the C-17 FSM does) doesn't seem to carry the high cost we expected!

There are a few things which, based on hindsight, I might have done differently. First of all, in Sections 3-6 and 4-2, I would have used a log-linear learning curve data set

which had no error in it. (The model we used had a uniformly distributed error term with a *very* small variance.) Using data which had no error term may have provided a more useful initial analysis since the models would have had a 'clean' data set for the initial test instead of having to deal with the random component injected by the error term. Of course, the error we used in the model was quite small and may have had a negligible effect anyway.

Another thing I might have done differently was to use all of the models to fit/forecast the log-linear learning curve data the same way we used them to fit/forecast the F-102 data and the Notional C-17 data; this would have provided a more complete picture of the abilities of the candidate models. As it was, in Section 4-2, we used the models only to fit the log-linear data; in Section 4-3, we used the models to fit the F-102 data and then went on and used them to fit/forecast the F-102 data; in Section 4-4, we used the models only to fit/forecast the Notional C-17 data. Each section should have used precisely the same set of investigations.

Yet another modification I might make to my investigations would be to have picked a sample size for the fit/forecasting investigations which was common to all data sets. For example, since the Notional C-17 data fit/forecasting investigations used a sample of 15 observations on which to base the fitting of the models, we should have used this same number in the fit/forecasting of the F-102 data; furthermore, if our investigations had included fit/forecasting for the log-linear data, this number should also have been the same for them. Using different numbers of observations in the fit part of the

fit/forecasting, put the candidate models on uneven ground and make it difficult to evaluate the results of the study.

One more thing I would do if there was more time, would be to take a closer look at the SSE's (used as measures of performance in Chapter 4). It would be interesting to see how close in magnitude each of the resulting SSE's are. Perhaps we'd find that there is virtually no difference between the fidelity of the top four models; we might find that any one of them will do an equally good job at forecasting so we could choose the lowest order or simplest model from among them in pursuit of parsimony or simplicity. On the other hand, we might find that the model ranked number one had a statistic which was six orders of magnitude better than the next best model's statistic; this might *keep* us from choosing a lower order model. I feel that the relative magnitudes of the SSE's is quite an important consideration in the analysis of the results.

The point of this thesis was <u>not</u> to show that log-linear learning curve models are ~~no~~ *not* good at forecasting learning type data, that ARMA models were the best models for this type of forecasting, or that the C-17 FSM should be scrapped because it failed to account for uncertainty in fit. Instead, the objective was to show that there are other models which provide a viable alternative to the standard learning curve. The secondary emphasis was to investigate the merit/cost of ignoring, within the C-17 FSM, the uncertainty in fit. I believe I've accomplished both of these things! Hopefully, model developers can utilize some of the information in this thesis to assist them in developing the best forecasting models possible.

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Appendix A

Forecast-Pro Summary and Worksheets
(Work done in Forecast-Pro.)

Table A-1 Forecast-Pro, Summary of Statistics and Parameters

	RSQR	ADJRSQ	Dubin-Watson	MAPE	MAD	BIC	RMSE
Simple Exp. Smoothing	0.9382	0.9382	0.9442	0.02687	14.53	36.47	35.07
SMA(1)	0.9022	0.9042	0.2607	0.02742	14.83	35.43	35.43
Holt Exp. Smoothing	-	-	-	-	-	-	-
AR(1)	0.9401	0.9401	0.9888	0.02714	14.58	35.91	34.54
AR(2)	0.9841	0.9838	2.04	0.01623	8.227	19.24	17.79
AR(3)	0.9833	0.9826	1.686	0.01657	8.382	20.53	18.25
AR(4)	0.9844	0.9833	1.733	0.01603	8.023	20.64	17.65
MA(1)	0.8037	0.8037	0.05782	0.1155	47.34	65.02	62.52
MA(2)	0.9254	0.9239	0.5625	0.07117	28.74	41.67	38.53
MA(3)	0.9723	0.9711	0.8157	0.04995	19.54	26.42	23.49
MA(4)	0.9872	0.9863	1.09	0.03225	12.63	18.69	15.98
ARMA(1,1)	0.9646	0.9639	0.8359	0.02662	13.28	28.7	26.54
ARMA(2,1)	0.9865	0.9859	2.348	0.0181	8.177	18.45	16.4
ARMA(1,2)	0.9908	0.9904	1.73	0.01687	8.051	15.23	13.54
ARMA(2,2)	0.9893	0.9886	2.207	0.01859	8.357	17.08	14.61

	Level(wt,val)	Trend(wt,val)	a[1]	S	a[2]	S	a[3]
Simple Exp. Smoothing	1286.22	-	-	-	-	-	-
SMA(1)	-	-	-	-	-	-	-
Holt Exp. Smoothing	.99973, 286.22	.23092, -1.3716	-	-	-	-	-
AR(1)	-	-	0.9953	1	-	-	-
AR(2)	-	-	1.7947	1	-0.8018	1	-
AR(3)	-	-	1.5845	1	-0.2266	0.97	-0.3632
AR(4)	-	-	1.6203	1	-0.2361	0.9347	-0.4572
MA(1)	-	-	-	-	-	-	-
MA(2)	-	-	-	-	-	-	-
MA(3)	-	-	-	-	-	-	-
MA(4)	-	-	-	-	-	-	-
ARMA(1,1)	-	-	0.9954	1	-	-	-
ARMA(2,1)	-	-	1.8974	1	-0.9048	1	-
ARMA(1,2)	-	-	0.9991	1	-	-	-
ARMA(2,2)	-	-	1.8956	1	-0.9072	1	-

	S	a[4]	S	b[1]	S	b[2]	S
Simple Exp. Smoothing	-	-	-	-	-	-	-
SMA(1)	-	-	-	-	-	-	-
Holt Exp. Smoothing	-	-	-	-	-	-	-
AR(1)	-	-	-	-	-	-	-
AR(2)	-	-	-	-	-	-	-
AR(3)	1	-	-	-	-	-	-
AR(4)	0.976	0.0676	0.5	-	-	-	-
MA(1)	-	-	-	-0.9675	1	-	-
MA(2)	-	-	-	-1.3321	1	-0.9397	1
MA(3)	-	-	-	-1.5788	1	-1.528	1
MA(4)	-	-	-	-1.7923	1	-2.2399	1
ARMA(1,1)	-	-	-	-0.8783	1	-	-
ARMA(2,1)	-	-	-	-0.249	0.9768	-	-
ARMA(1,2)	-	-	-	-1.1743	1	-0.9597	1
ARMA(2,2)	-	-	-	-0.4297	0.9998	-0.5006	1

	b[3]	S	b[4]	S	Const
Simple Exp. Smoothing	-	-	-	-	-
SMA(1)	-	-	-	-	-
Holt Exp. Smoothing	-	-	-	-	-
AR(1)	-	-	-	-	1.9008
AR(2)	-	-	-	-	2.8753
AR(3)	-	-	-	-	2.1476
AR(4)	-	-	-	-	2.1717
MA(1)	-	-	-	-	402.22
MA(2)	-	-	-	-	402.22
MA(3)	-0.8981	1	-	-	402.22
MA(4)	-1.6646	1	-0.8566	1	402.22
ARMA(1,1)	-	-	-	-	1.8609
ARMA(2,1)	-	-	-	-	2.9941
ARMA(1,2)	-	-	-	-	0.3623
ARMA(2,2)	-	-	-	-	4.6625

Forecast Pro for Windows Standard Edition Version 2.00
Mon Jan 29 19:55:18 1996

Expert data exploration of dependent variable PWRMEANS

Length 50 Minimum 283.069 Maximum 1002.019
Mean 402.219 Standard deviation 141.123

Series too short to determine seasonality. Treating as nonseasonal.
Classical decomposition (nonseasonal)
Trend-cycle: 97.11% Irregular: 2.89%

Log transform recommended for Box-Jenkins.

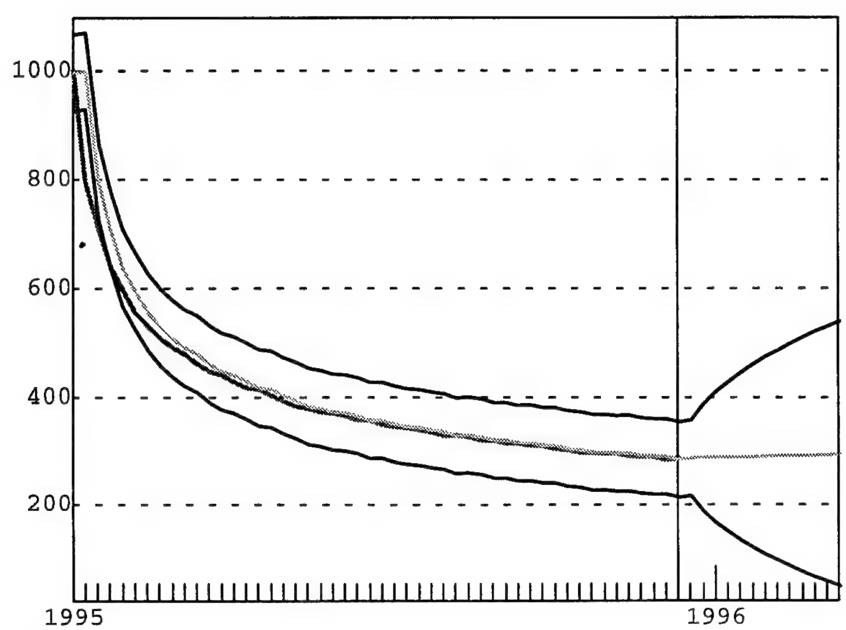
There are no strongly significant regressors, so I will choose a univariate method.

Exponential smoothing outperforms Box-Jenkins by 2.588 to 3.628 out-of-sample (MAD). I tried 21 forecasts up to a maximum horizon 6.
For Box-Jenkins, I used a log transform.

Series is trended and nonseasonal.

Recommended model: Exponential Smoothing

Figure A-1 Forecast-Pro, Exponential Smoothing



Simple exponential smoothing

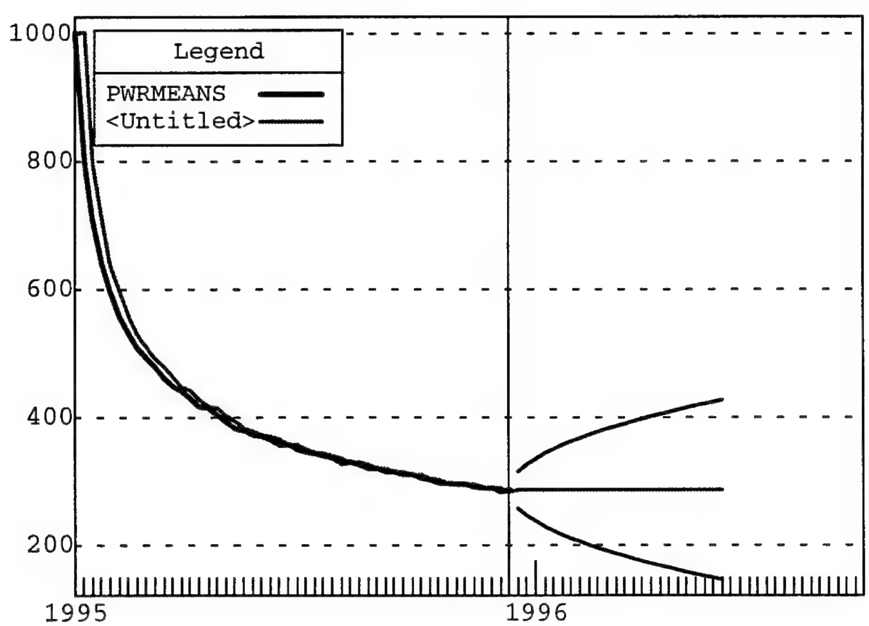
Forecast Model for PWRMEANS
Simple exponential smoothing: No trend, No seasonality
Confidence limits proportional to level

Component	Smoothing Weight	Final Value
Level	1.00000	286.22

Standard Diagnostics

Sample size 50	Number of parameters 1
Mean 402.2	Standard deviation 142.6
R-square .9954	Adjusted R-square 0.9382
Durbin-Watson 0.9442	** Ljung-Box(18)=42.45 P=0.999
Forecast error 35.43	BIC 36.47
MAPE 0.02687	RMSE 35.07
MAD 14.53	

Figure A-2 Forecast-Pro, Simple Exponential Smoothing



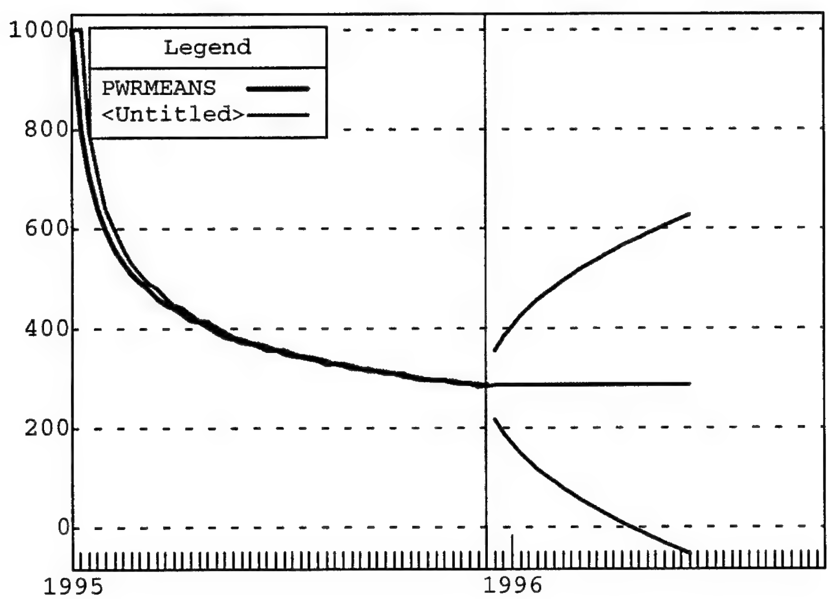
Naive (SMA(1), Random walk)

Forecast Model for PWRMEANS
Automatic model selection
Naive (SMA(1), Random walk)

Standard Diagnostics

Sample size 49	Number of parameters 0
Mean 390	Standard deviation 114.4
R-square 0.9022	Adjusted R-square 0.9042
Durbin-Watson 0.2607	** Ljung-Box(18)=41.51 P=0.9987
Forecast error 35.43	BIC 35.43
MAPE 0.02742	RMSE 35.43
MAD 14.83	

Figure A-3 Forecast-Pro, Simple Moving Average (SMA(1))



Holt exponential smoothing

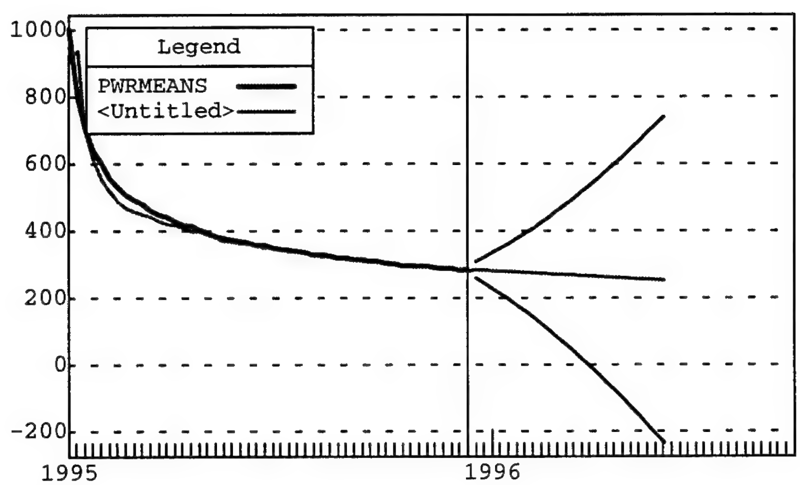
Forecast Model for PWRMEANS
Automatic model selection
Holt exponential smoothing: Linear trend, No seasonality
Confidence limits proportional to level

Component	Smoothing Weight	Final Value
Level	0.99973	286.22
Trend	0.23092	-1.3716

Standard Diagnostics

Sample size 50	Number of parameters 2
Mean 402.2	Standard deviation 142.6
R-square 0.9611	Adjusted R-square 0.9603
Durbin-Watson 1.938	Ljung-Box(18)=6.991 P=0.009796
Forecast error 28.39	BIC 30.08 (Best so far)
MAPE 0.02575	RMSE 27.82
MAD 13.83	

Figure A-4 Forecast-Pro, Holt Exponential Smoothing



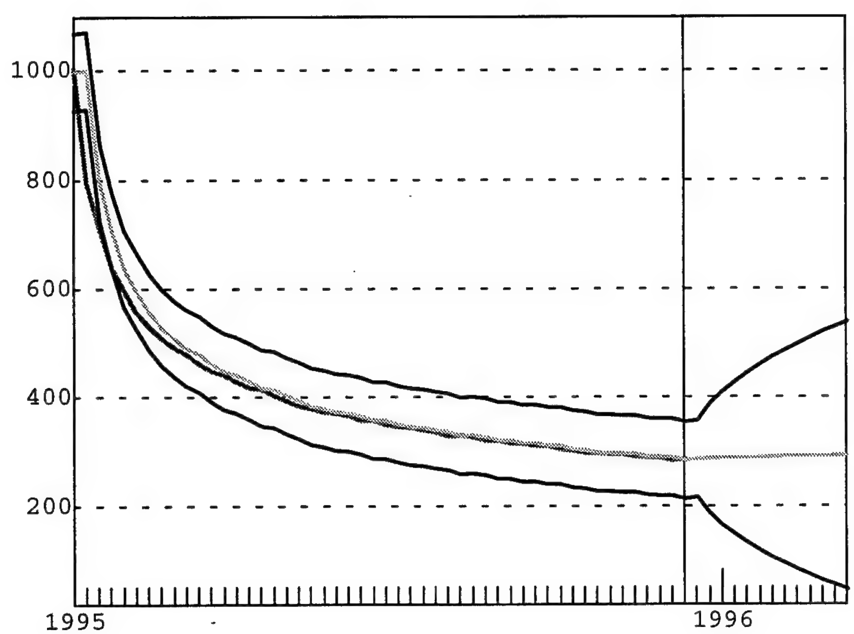
Forecast Model for PWRMEANS
ARIMA(1,0,0)

Term	Coefficient	Std. Error	t-Statistic	Significance
a[1]	0.9953	0.0056	178.7528	1.0000
_CONST	1.9008			

Standard Diagnostics

Sample size 50	Number of parameters 1
Mean 402.2	Standard deviation 142.6
R-square 0.9401	Adjusted R-square 0.9401
Durbin-Watson 0.9888	** Ljung-Box(18)=40.21 P=0.998
Forecast error 34.89	BIC 35.91
MAPE 0.02714	RMSE 34.54
MAD 14.58	

Figure A-5 Forecast-Pro, AR(1)



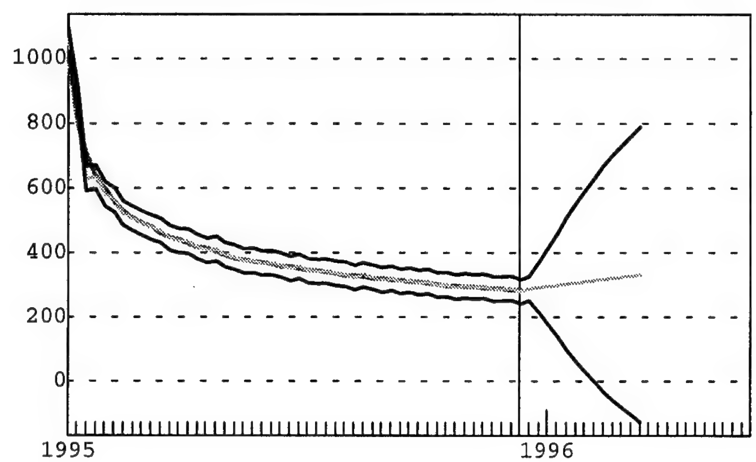
Forecast Model for PWRMEANS
ARIMA(2,0,0)

Term	Coefficient	Std. Error	t-Statistic	Significance
a[1]	1.7947	0.0479	37.4544	1.0000
a[2]	-0.8018	0.0494	-16.2365	1.0000
_CONST	2.8753			

Standard Diagnostics

Sample size 50	Number of parameters 2
Mean 402.2	Standard deviation 142.6
R-square 0.9841	Adjusted R-square 0.9838
Durbin-Watson 2.04	Ljung-Box(18)=5.466 P=0.002073
Forecast error 18.16	BIC 19.24 (Best so far)
MAPE 0.01623	RMSE 17.79
MAD 8.227	

Figure A-6 Forecast-Pro, AR(2)



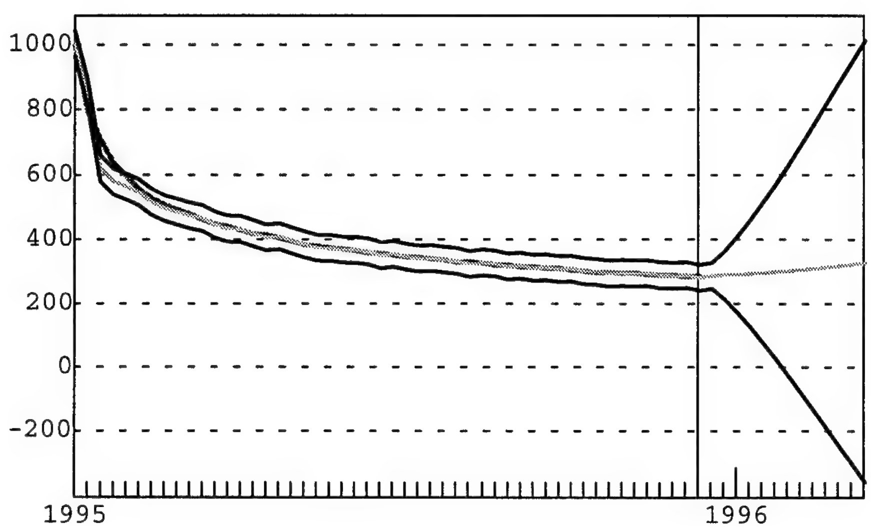
Forecast Model for PWRMEANS
ARIMA(3,0,0)

Term	Coefficient	Std. Error	t-Statistic	Significance
a[1]	1.5845	0.0448	35.3651	1.0000
a[2]	-0.2266	0.0986	-2.2984	0.9740
a[3]	-0.3632	0.0556	-6.5293	1.0000
_CONST	2.1476			

Standard Diagnostics

Sample size 50	Number of parameters 3
Mean 402.2	Standard deviation 142.6
R-square 0.9833	Adjusted R-square 0.9826
Durbin-Watson 1.686	Ljung-Box(18)=3.235 P=4.927e-005
Forecast error 18.83	BIC 20.53
MAPE 0.01657	RMSE 18.25
MAD 8.382	

Figure A-7 Forecast-Pro, AR(3)



Forecast Model for PWRMEANS
ARIMA(4,0,0)

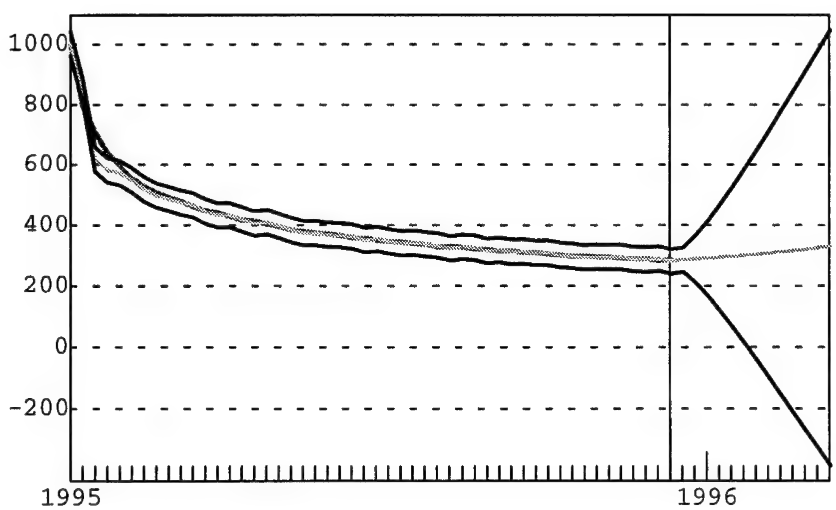
Term	Coefficient	Std. Error	t-Statistic	Significance
a[1]	1.6203	0.0414	39.1669	1.0000
a[2]	-0.2361	0.1250	-1.8883	0.9347
a[3]	-0.4572	0.1958	-2.3350	0.9760
a[4]	0.0676	0.0995	0.6791	0.4995
_CONST	2.1717			

Try alternative model ARIMA(3,0,0)

Standard Diagnostics

Sample size 50	Number of parameters 4
Mean 402.2	Standard deviation 142.6
R-square 0.9844	Adjusted R-square 0.9833
Durbin-Watson 1.733	Ljung-Box(18)=3.237 P=4.951e-005
Forecast error 18.4	BIC 20.64
MAPE 0.01603	RMSE 17.65
MAD 8.023	

Figure A-8 Forecast-Pro, AR(4)



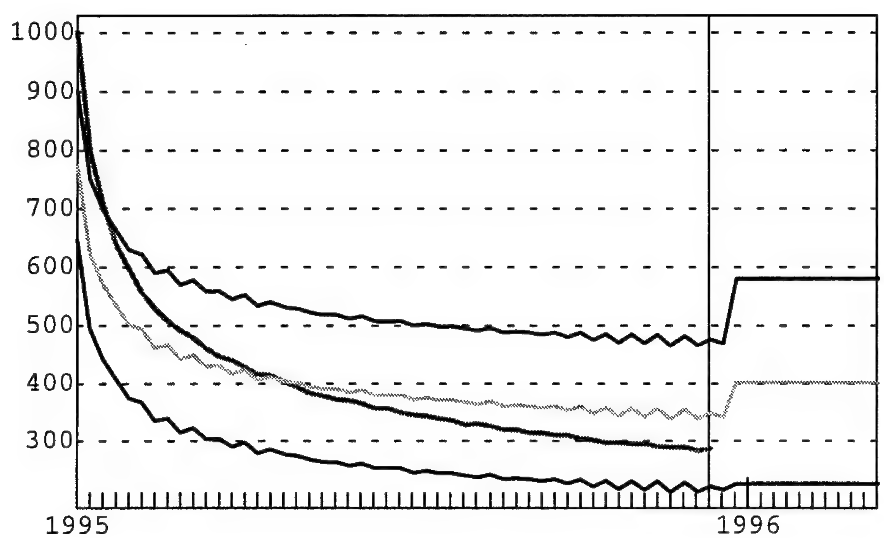
Forecast Model for PWRMEANS
ARIMA(0,0,1)

Term	Coefficient	Std. Error	t-Statistic	Significance
b[1]	-0.9675	0.0200	-48.4850	1.0000
_CONST	402.2187			

Standard Diagnostics

Sample size 50	Number of parameters 1
Mean 402.2	Standard deviation 142.6
R-square 0.8037	Adjusted R-square 0.8037
Durbin-Watson 0.05782	** Ljung-Box(18)=158.7 P=1
Forecast error 63.16	BIC 65.02
MAPE 0.1155	RMSE 62.52
MAD 47.34	

Figure A-9 Forecast-Pro, MA(1)



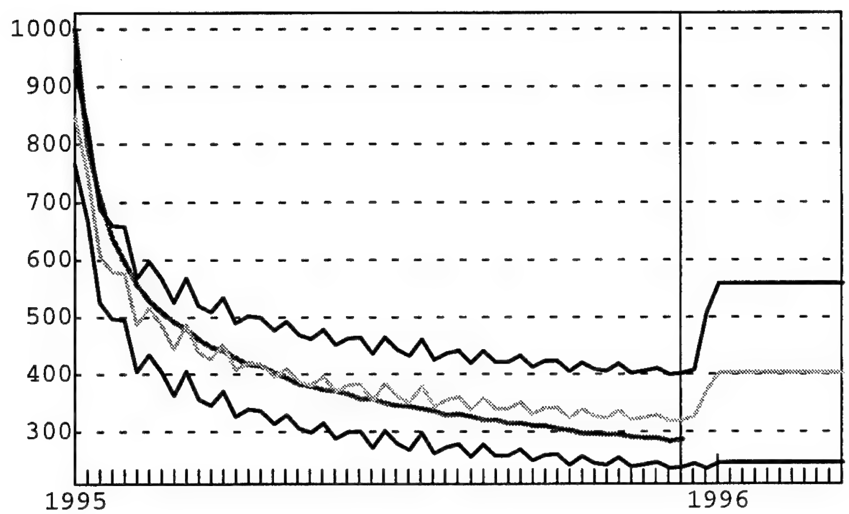
Forecast Model for PWRMEANS
ARIMA(0,0,2)

Term	Coefficient	Std. Error	t-Statistic	Significance
b[1]	-1.3321	0.0563	-23.6572	1.0000
b[2]	-0.9397	0.0413	-22.7603	1.0000
_CONST	402.2187			

Standard Diagnostics

Sample size 50	Number of parameters 2
Mean 402.2	Standard deviation 142.6
R-square 0.9254	Adjusted R-square 0.9239
Durbin-Watson 0.5625	** Ljung-Box(18)=121.6 P=1
Forecast error 39.33	BIC 41.67
MAPE 0.07117	RMSE 38.53
MAD 28.74	

Figure A-10 Forecast-Pro, MA(2)



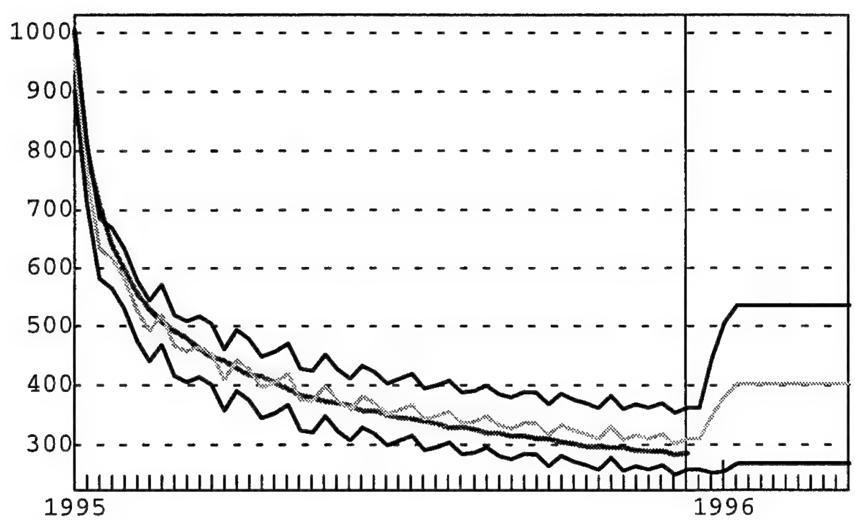
Forecast Model for PWRMEANS
ARIMA(0,0,3)

Term	Coefficient	Std. Error	t-Statistic	Significance
b[1]	-1.5788	0.0562	-28.0915	1.0000
b[2]	-1.5280	0.0661	-23.1201	1.0000
b[3]	-0.8981	0.0410	-21.8820	1.0000
_CONST	402.2187			

Standard Diagnostics

Sample size 50	Number of parameters 3
Mean 402.2	Standard deviation 142.6
R-square 0.9723	Adjusted R-square 0.9711
Durbin-Watson 0.8157	** Ljung-Box(18)=128.3 P=1
Forecast error 24.23	BIC 26.42
MAPE 0.04995	RMSE 23.49
MAD 19.54	

Figure A-11 Forecast-Pro, MA(3)



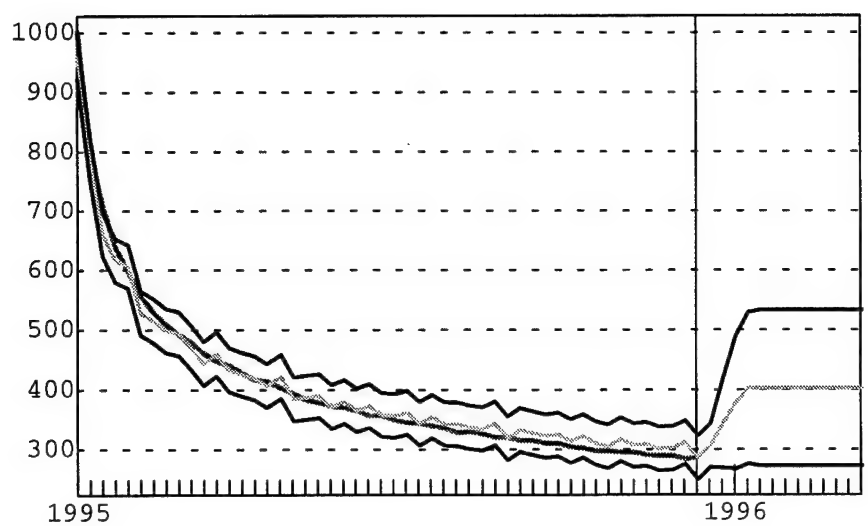
Forecast Model for PWRMEANS
ARIMA(0,0,4)

Term	Coefficient	Std. Error	t-Statistic	Significance
b[1]	-1.7923	0.0546	-32.7994	1.0000
b[2]	-2.2399	0.0883	-25.3794	1.0000
b[3]	-1.6646	0.0802	-20.7439	1.0000
b[4]	-0.8566	0.0452	-18.9600	1.0000
_CONST	402.2187			

Standard Diagnostics

Sample size 50	Number of parameters 4
Mean 402.2	Standard deviation 142.6
R-square 0.9872	Adjusted R-square 0.9863
Durbin-Watson 1.09	** Ljung-Box(18)=107.4 P=1
Forecast error 16.66	BIC 18.69 (Best so far)
MAPE 0.03225	RMSE 15.98
MAD 12.63	

Figure A-12 Forecast-Pro, MA(4)



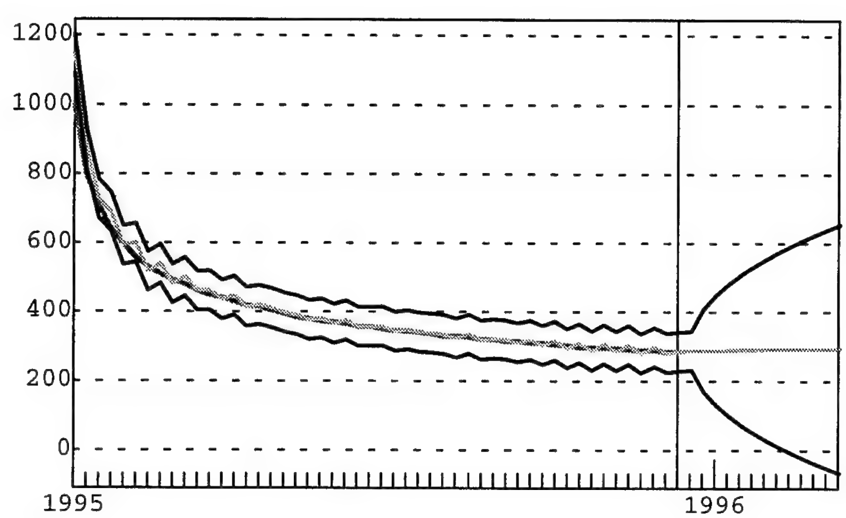
Forecast Model for PWRMEANS
ARIMA(1,0,1)

Term	Coefficient	Std. Error	t-Statistic	Significance
a[1]	0.9954	0.0061	163.9297	1.0000
b[1]	-0.8783	0.0604	-14.5466	1.0000
_CONST	1.8609			

Standard Diagnostics

Sample size 50	Number of parameters 2
Mean 402.2	Standard deviation 142.6
R-square 0.9646	Adjusted R-square 0.9639
Durbin-Watson 0.8359	Ljung-Box(18)=20.31 P=0.6841
Forecast error 27.09	BIC 28.7
MAPE 0.02662	RMSE 26.54
MAD 13.28	

Figure A-13 Forecast-Pro, ARMA(1,1)



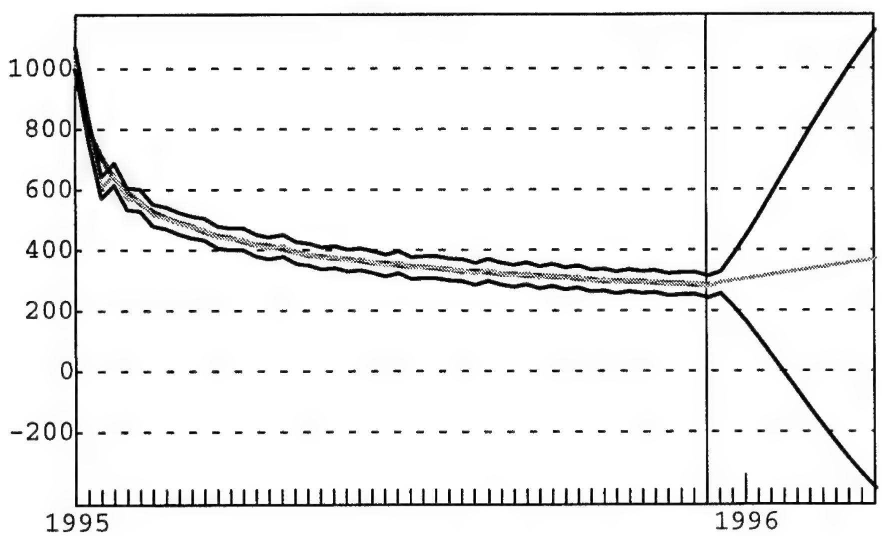
Forecast Model for PWRMEANS
ARIMA(2,0,1)

Term	Coefficient	Std. Error	t-Statistic	Significance
a[1]	1.8974	0.0387	49.0106	1.0000
a[2]	-0.9048	0.0403	-22.4327	1.0000
b[1]	-0.2490	0.1061	-2.3476	0.9768
_CONST	2.9941			

Standard Diagnostics

Sample size 50	Number of parameters 3
Mean 402.2	Standard deviation 142.6
R-square 0.9865	Adjusted R-square 0.9859
Durbin-Watson 2.348	Ljung-Box(18)=6.226 P=0.004808
Forecast error 16.92	BIC 18.45 (Best so far)
MAPE 0.0181	RMSE 16.4
MAD 8.177	

Figure A-14 Forecast-Pro, ARMA(2,1)



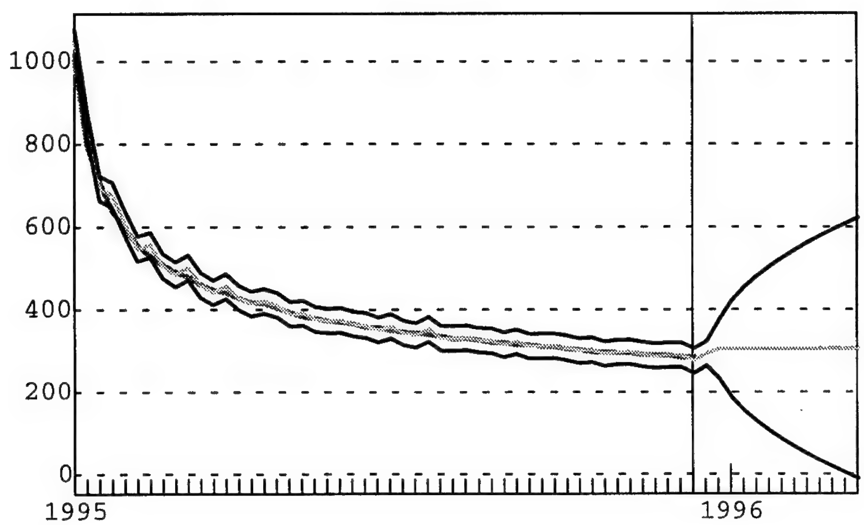
Forecast Model for PWRMEANS
ARIMA(1,0,2)

Term	Coefficient	Std. Error	t-Statistic	Significance
a[1]	0.9991	0.0041	241.5930	1.0000
b[1]	-1.1743	0.0591	-19.8619	1.0000
b[2]	-0.9597	0.0334	-28.7005	1.0000
_CONST	0.3623			

Standard Diagnostics

Sample size 50	Number of parameters 3
Mean 402.2	Standard deviation 142.6
R-square 0.9908	Adjusted R-square 0.9904
Durbin-Watson 1.73	Ljung-Box(18)=28.68 P=0.9476
Forecast error 13.97	BIC 15.23 (Best so far)
MAPE 0.01687	RMSE 13.54
MAD 8.051	

Figure A-15 Forecast-Pro, ARMA(1,2)



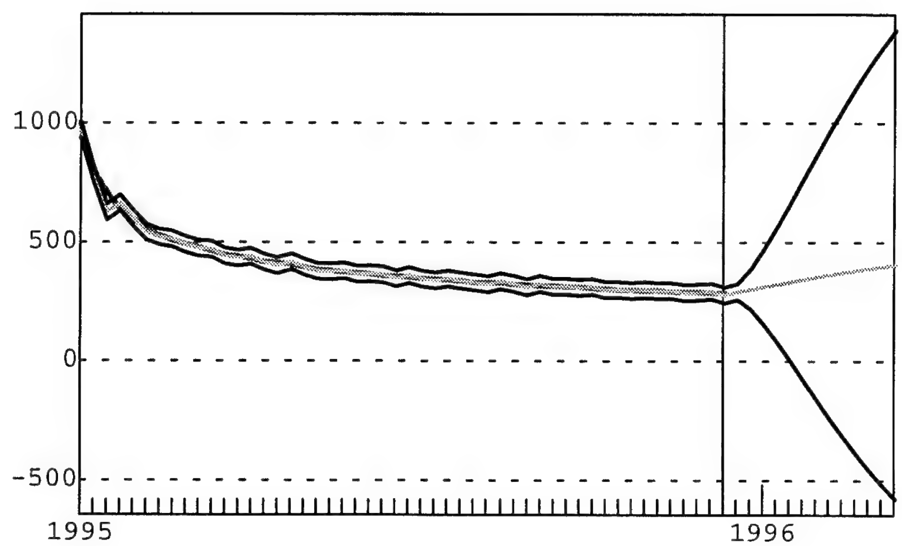
Forecast Model for PWRMEANS
ARIMA(2,0,2)

Term	Coefficient	Std. Error	t-Statistic	Significance
a[1]	1.8956	0.0462	41.0309	1.0000
a[2]	-0.9072	0.0468	-19.3879	1.0000
b[1]	-0.4297	0.1050	-4.0940	0.9998
b[2]	-0.5006	0.1039	-4.8165	1.0000
_CONST	4.6625			

Standard Diagnostics

Sample size 50	Number of parameters 4
Mean 402.2	Standard deviation 142.6
R-square 0.9893	Adjusted R-square 0.9886
Durbin-Watson 2.207	Ljung-Box(18)=14.53 P=0.3059
Forecast error 15.23	BIC 17.08 (Best so far)
MAPE 0.01859	RMSE 14.61
MAD 8.357	

Figure A-16 Forecast-Pro, ARMA(2,2)



Appendix B

Fitting the Simulated Log-Linear Learning Curve Data Using 50 Observations
(Work done in EXCEL)

Table B-1 Detailed Summary of Log-Linear Fitting

Fitting the Log-Linear Data				
Log-Linear Model				
Parameters:	**a**	**b***		
	1004.333179	-0.323752095		
SSE:	763.686			
Equation:	**Y(x)=a*x^b**			
Forsythe				
Parameters:	**a***	**b***	**cmin**	
	929.17375	-0.402443468	100	
SSE:	**2.256E+03**	0	0	
Equation:	**Y(x)=a*x^b+cmin**			
Stanford-B				
Parameters:	**a***	**beta***	**n***	
	1217.51483	1	-0.38003	
SSE:	**1.158E+04**			
Equation:	**Y(x)=a*(x+beta)^n**			
Pegel				
Parameters:	**alpha**	**a**	**beta**	
	665.37197	0.977241524	0.022758477	
SSE:	**2.427E+05**	1		
Equation:	**MC(x)=alpha*a^(x-1)+beta**			
S-Curve				
Parameters:	**a**	**L***	**b***	**k***
	5036.25768	4718.94251	0.150662349	0.149861837
SSE:	**4.324E+04**	0		
Equation:	**Y(x)=L*exp(-b*exp(-k*t))**			
AR(1)				
Parameters:	**phi***	**const***		
	0.78917	70.55227		
SSE:	**9.686E+03**			
Equation:	**AR(1)= phi*(AR1-1)+const+noise**			
AR(2)				
Parameters:	**phi_1***	**phi_2***	**const***	
	0.74968	0.07484	56.68531	
SSE:	**4.851E+03**			
Equation:	**AR(2)= phi1*(AR2-1)+phi2*(AR2-2)+const+noise**			

(more) Table B-1 EXCEL, Detailed Summary of Log-Linear Fitting

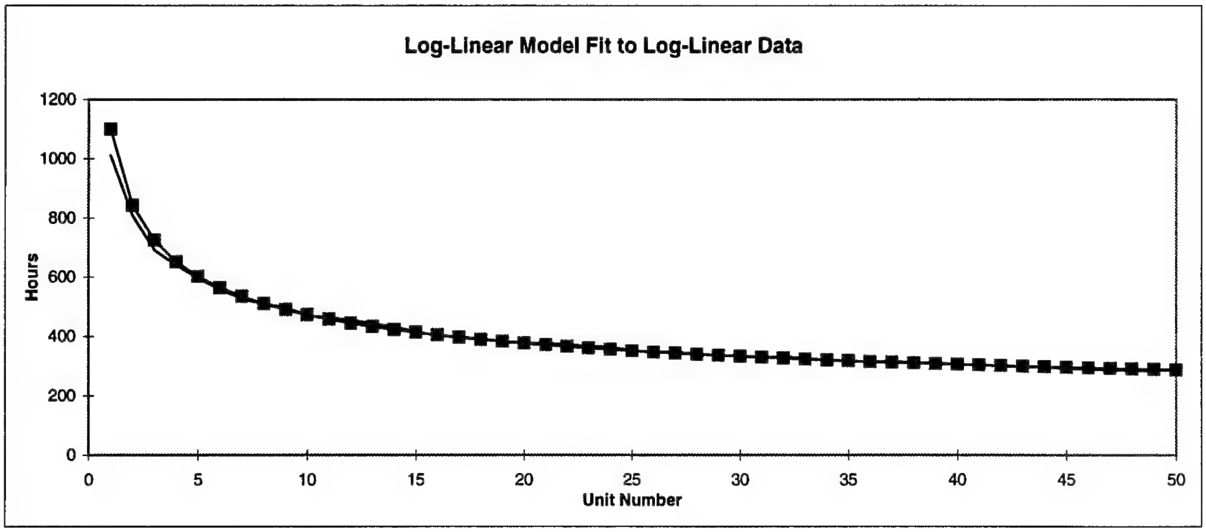
AR(3)					
Parameters:	**phi_1***	**phi_2***	**phi_3***	**const***	
	0.75637	0.03694	0.05926	45.79889	
SSE:	**1.756E+03**				
Equation:	**AR(3)= phi1*(AR3-1)+phi2*(AR3-2)+phi3*(AR3-3)+ const+noise**				
AR(4)					
Parameters:	**phi_1***	**phi_2***	**phi_3***	**phi_4***	**const***
	0.75873	0.03707	0.04687	0.01883	42.37111
SSE:	**1.439E+03**				
Equation:	**AR(4)= phi1*(AR4-1)+phi2*(AR4-2)+phi3*(AR4-3)+ phi4*(AR4-4)+const+noise**				
MA(1)					
Parameters:	**myou***	**theta***			
	455.64949	0.471978149			
SSE:	**2.665E+06**				
Equation:	**MA(1)=myou-theta*errorminus1+noise**				
ARMA(1,1)					
Parameters:	**muprime***	**theta***	**phi***		
	88.77869	-0.77280	0.74083		
SSE:	**5.856E+03**				
Equation:	**ARMA(1,1)=phi*(ARMA(1,1)-1)+muprime-theta*errorlast+noise**				
ARMA(1,2)					
Parameters:	**muprime***	**phi1***	**theta1***	**theta2***	
	88.95685	0.74379	-0.61966	0.03737	
SSE:	**6.274E+03**				
Equation:	**ARMA(1,2)=phi1*(ARMA(1,2)-1)+muprime- theta1*errorlast-theta2*errorlastlast+noise**				
ARMA(2,1)					
Parameters:	**muprime***	**phi1***	**phi2***	**theta***	
	63.79179	0.76452	0.04179	-0.70546	
SSE:	**3.715E+03**				
Equation:	**ARMA(2,1) = phi1*(ARMA(2,1)-1)+phi2*(ARMA(2,1)-2)+ muprime-theta*errorlast+noise**				
ARMA(2,2)					
Parameters:	**muprime***	**phi1***	**phi2***	**theta1***	**theta2***
	62.50700	0.73840	0.07045	0.03757	-0.31920
SSE:	**4.468E+03**				
Equation:	**ARMA(2,2)=phi1*(ARMA(2,2)-1)+phi2*(ARMA(2,2)-2)+muprime- theta1*errorlast- theta2*errorlas**				

Table B-2 Brief Summary of Log-Linear Fitting

Model	SSE	Rank
Log-Linear	763.7	1
Forsythe	2256.4	4
Stanford-B	11576.1	11
Pegel	242678.9	13
S-Curve	43241.7	12
MA(1)	2665105.0	14
AR(1)	9685.7	10
AR(2)	4851.1	7
AR(3)	1756.4	3
AR(4)	1438.8	2
ARMA(1,1)	5856.5	8
ARMA(1,2)	6274.5	9
ARMA(2,1)	3714.9	5
ARMA(2,2)	4468.4	6

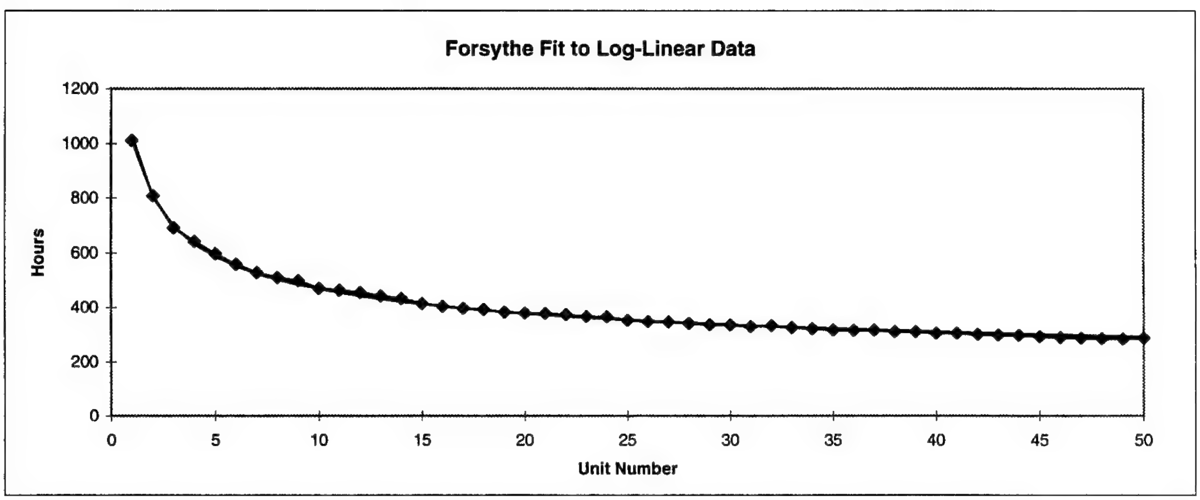
Parameters:	a	b*
	1004.333179	-0.323752095
SSE:	100.000	
Equation:	Y(x)=a*x^b	

Figure B-1 EXCEL, Log-Linear Fit to Log-Linear Data



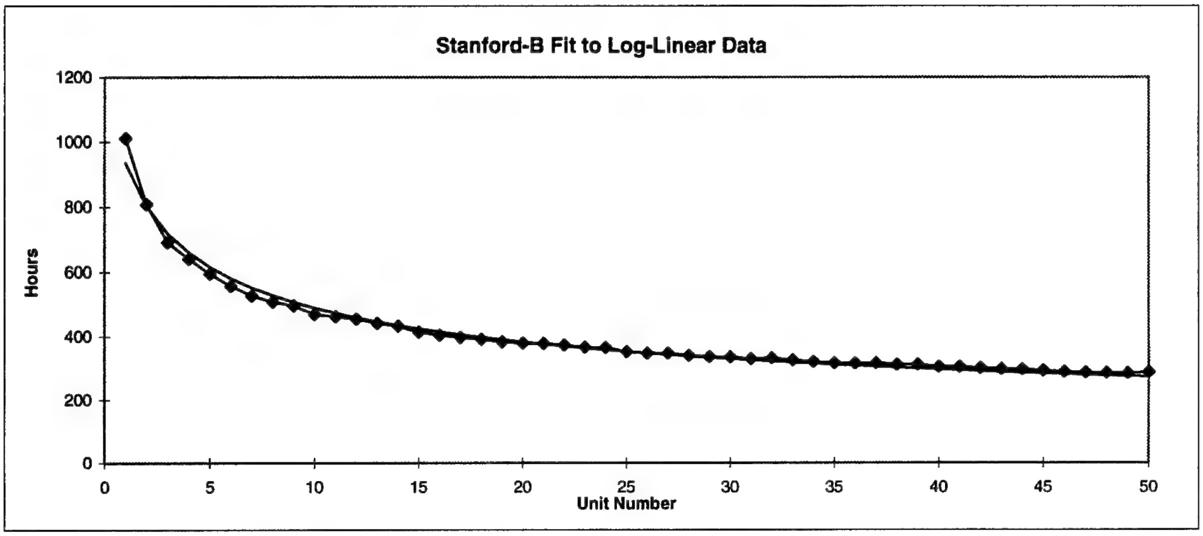
Parameters:	a*	b*	cmin
	929.1737539	-0.402443468	100
SSE:	2256.352		
Equation:	Y(x)=a*x^b+cmin		

Figure B-2 EXCEL, Forsythe Fit to Log-Linear Data



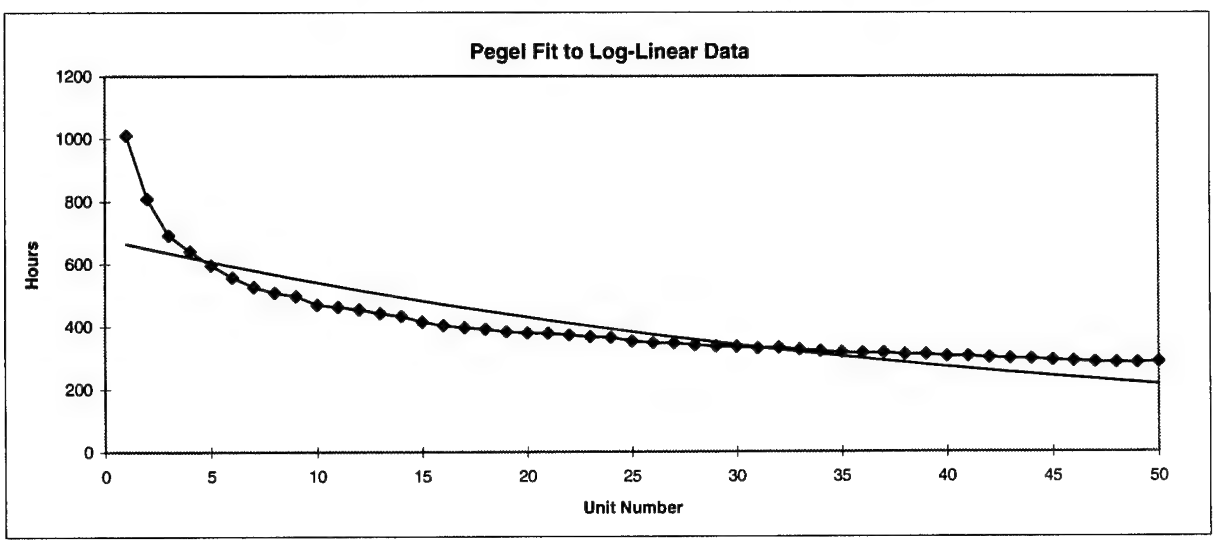
Parameters:	a*	beta*	n*
	1217.514829	1	-0.380034289
SSE:	11576.076		
Equation:	Y(x)=a*(x+beta)^n		

Figure B-3 EXCEL, Stanford-B Fit to Log-Linear Data



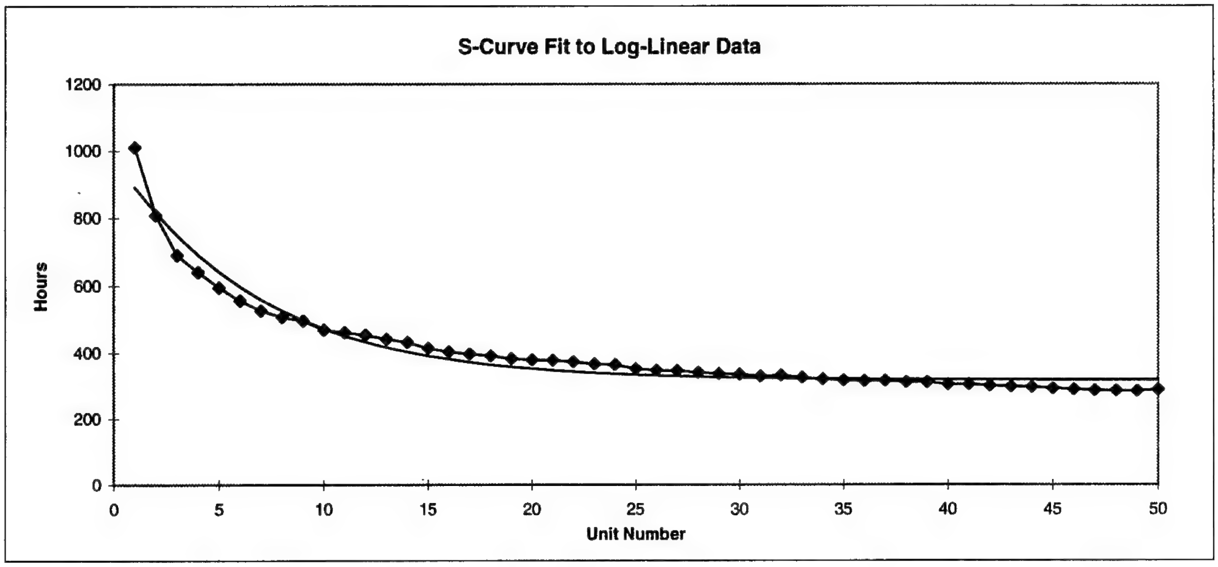
Parameters:	alpha*	a*	beta*
	665.3719671	0.977241524	0.022758477
SSE:	2.426789E+05	1	
Equation:	MC(x)=alpha*a^(x-1)+beta		

Figure B-4 EXCEL, Pegel's Fit to Log-Liner Data



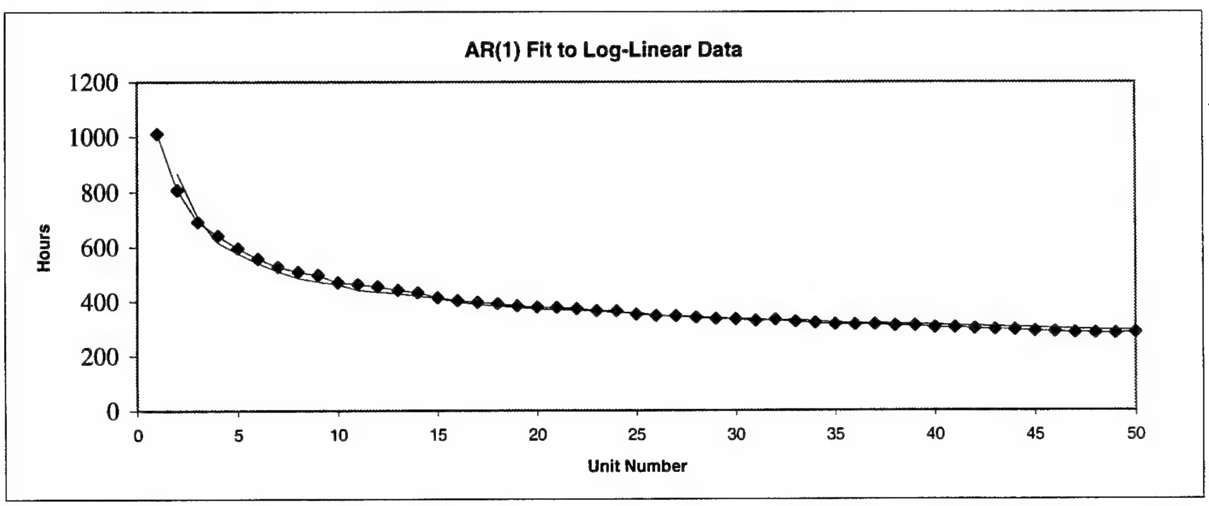
Parameters:	a*	L*	b*	k*
	5036.257676	4718.942513	0.150662349	0.149861837
SSE:	43241.67424	0		
Equation:	'(x)=L*exp(-b*exp(-k*t))			

Figure B-5 EXCEL, S-Curve Fit to Log-Linear Data



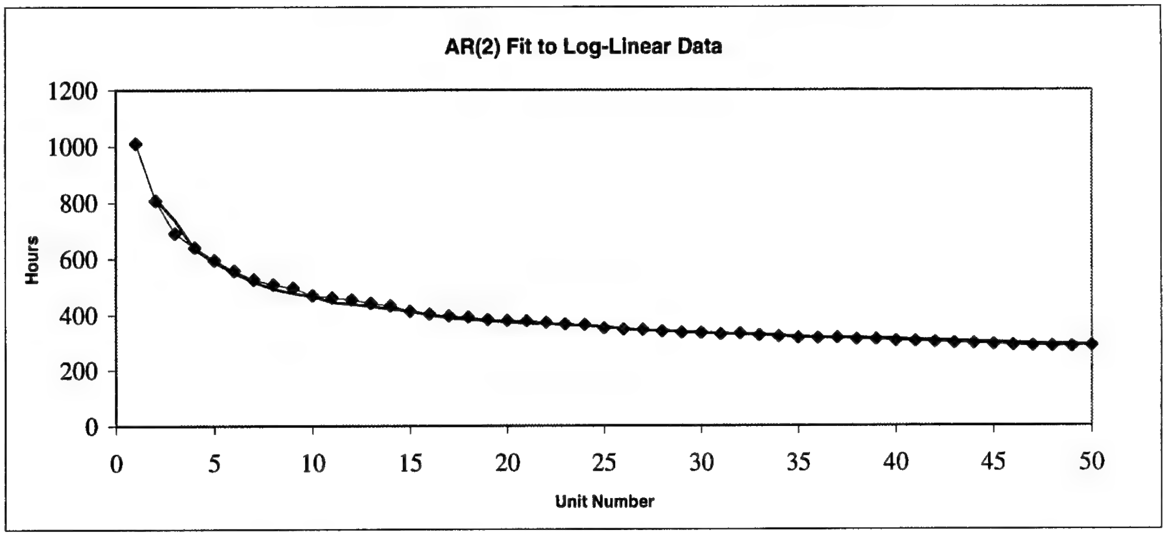
Parameters:	phi*	const*
	0.7892	70.5523
SSE:	9.686E+03	
Equation:	AR(1)= phi*(AR1-1)+const+noise	

Figure B-6 EXCEL, AR(1) Fit to Log-Linear Data



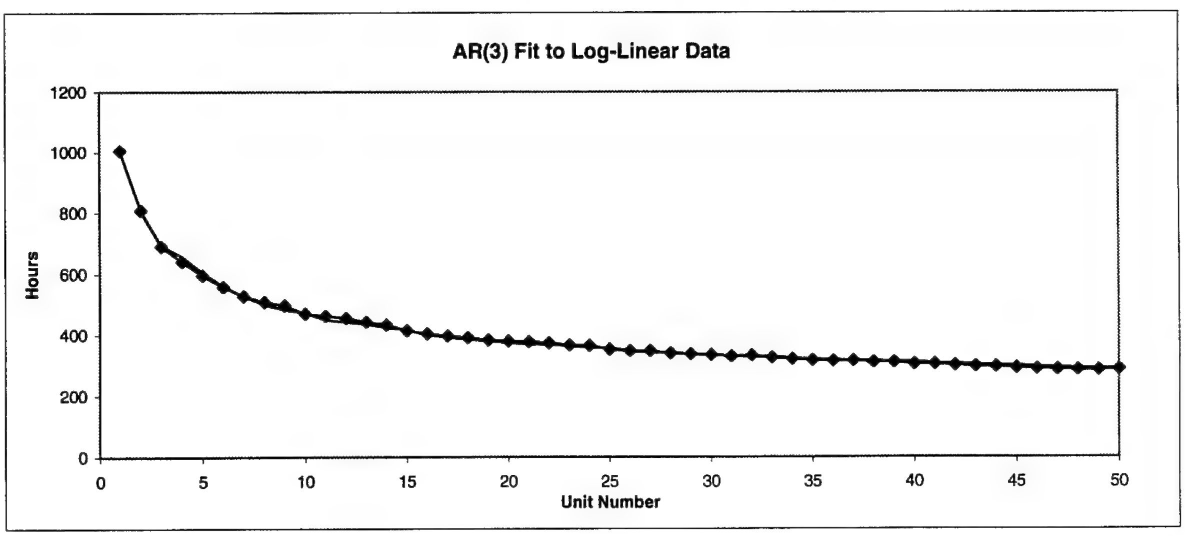
Parameters:	phi_1*	phi_2*	const*
	0.7497	0.074835731	56.6853
SSE:	4.851E+03		
Equation:	AR(2)= phi1*(AR2-1)+phi2*(AR2-2)+const+noise		

Figure B-7 EXCEL, AR(2) Fit to Log-Linear Data



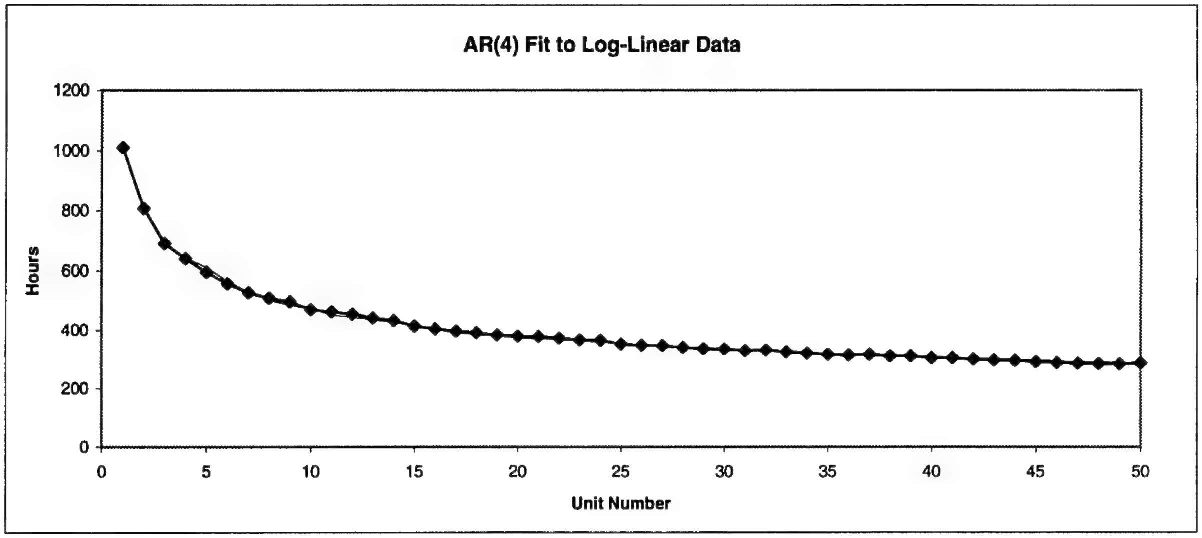
Parameters:	phi_1*	phi_2*	phi_3*	const*
	0.7564	0.036941845	0.059262984	45.7989
SSE:	1.756E+03			
Equation:	AR(3)= phi1*(AR3-1)+phi2*(AR3-2)+phi3*(AR3-3)+const+noise			

Figure B-8 EXCEL, AR(3) Fit to Log-Linear Data



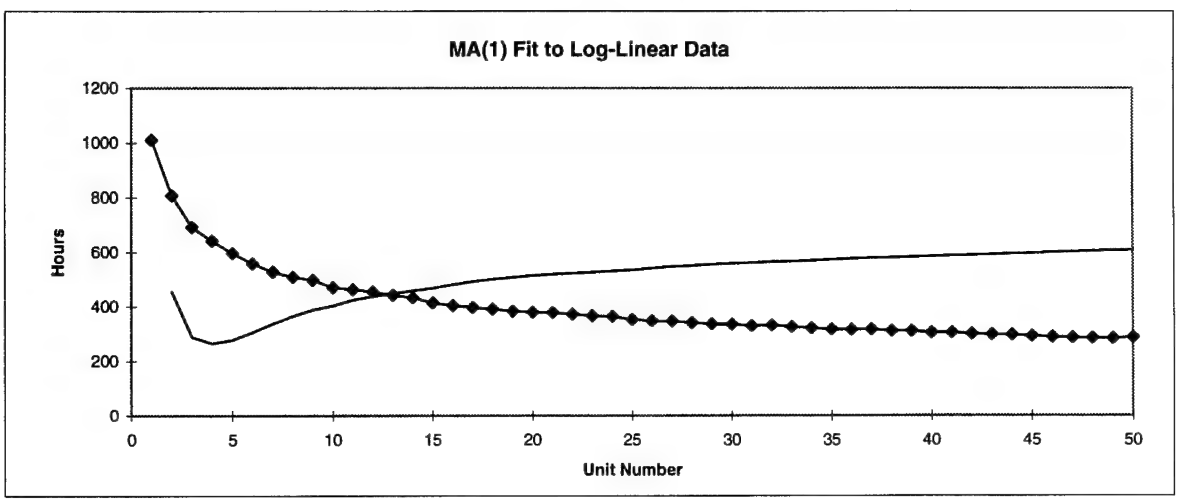
Parameters:	phi_1*	phi_2*	phi_3*	phi_4*	const*
	0.7587	0.037070476	0.046871928	0.01883124	42.3711
SSE:	1.439E+03				
Equation:	AR(4)= phi1*(AR4-1)+phi2*(AR4-2)+phi3*(AR4-3)+phi4*(AR4-4)+const+noise				

Figure B-9 EXCEL, AR(4) Fit to Log-Linear Data



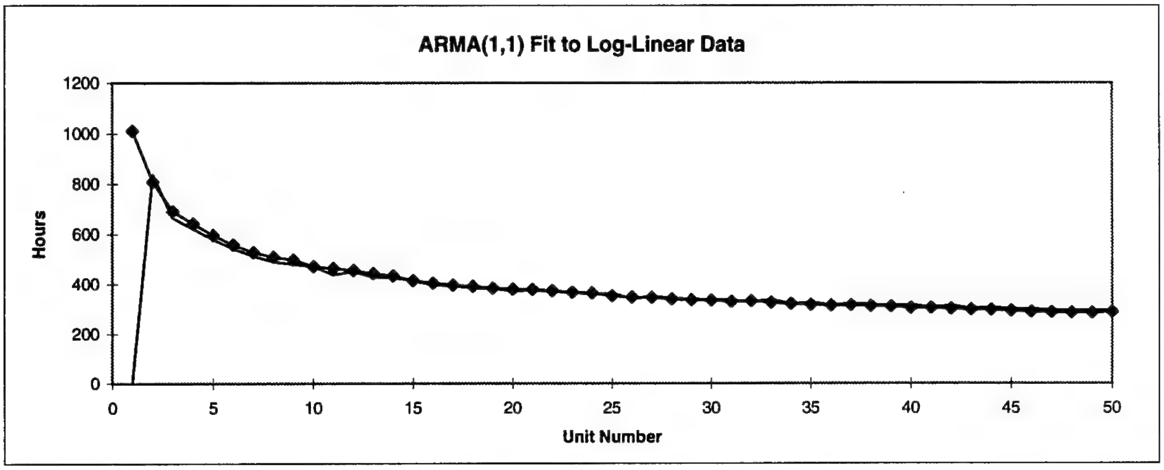
Parameters:	myou*	theta*
	455.6495	0.4720
SSE:	2.665E+06	
Equation:	MA(1)=myou-theta*errorminus1+noise	

Figure B-10 EXCEL, MA(1) Fit to Log-Liner Data



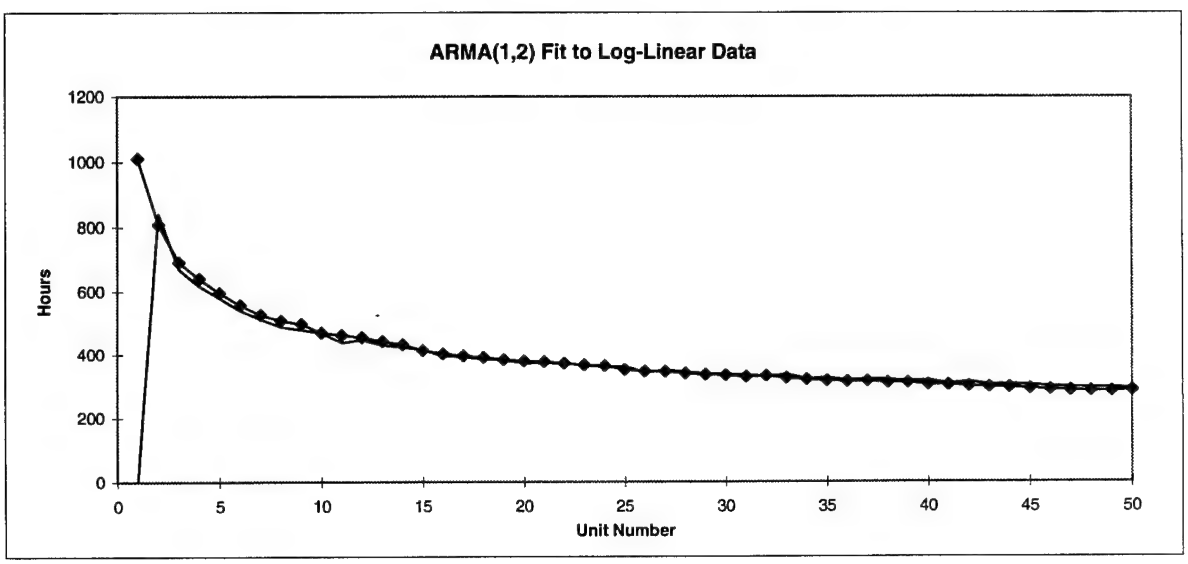
Parameters:	muprime*	theta*	phi*
	88.7787	-0.7728	0.7408
SSE:	5.856E+03		
Equation:	ARMA(1,1)=phi*(ARMA(1,1)-1)+muprime-theta*errorlast+noise		

Figure B-11 EXCEL, ARMA(1,1) Fit to Log-Linear Data



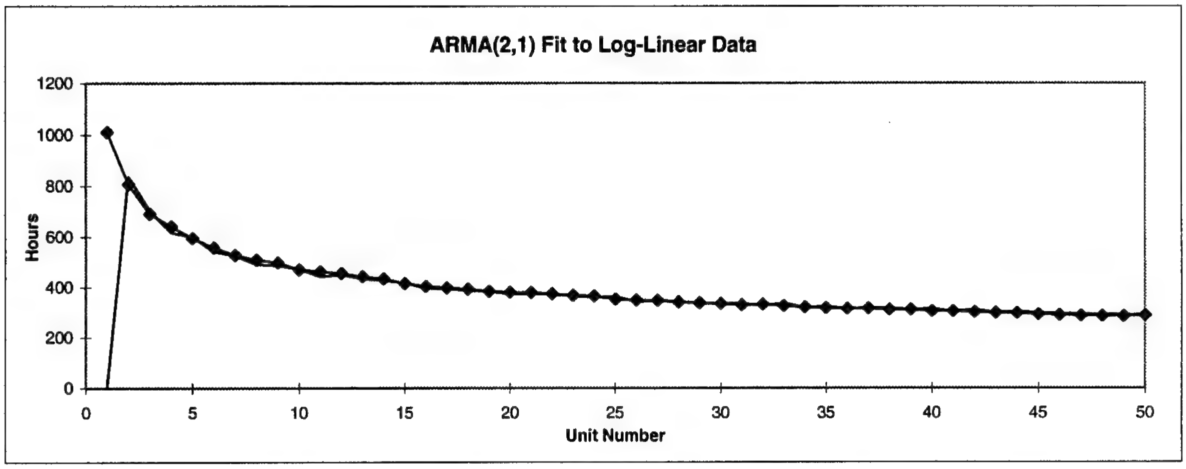
Parameters:	muprime*	phi1*	theta1*	theta2*
	88.9569	0.7438	-0.6197	0.037368591
SSE:	6.274E+03			
Equation:	ARMA(1,2)=phi1*(ARMA(1,2)-1)+muprime-theta1*errorlast-theta2*errorlastlast+noise			

Figure B-12 EXCEL, ARMA(1,2) Fit to Log-Linear Data



Parameters:	muprime*	phi1*	phi2*	theta*
	63.7918	0.7645	0.041791709	-0.7055
SSE:	3.715E+03			
Equation:	ARMA(2,1)=phi1*(ARMA(2,1)-1)+phi2*(ARMA(2,1)-2)+ muprime-theta*errorlast+noise			

Figure B-13 EXCEL, ARMA(2,1) Fit to Log-Linear Data



Parameters:	muprime*	phi1*	phi2*	theta1*	theta2*
	62.5070	0.7384	0.0704	0.037567463	-0.319203866
SSE:	4.468E+03				
Equation:	ARMA(2,2)=phi1*(ARMA(2,2)-1)+phi2*(ARMA(2,2)-2)+muprime- theta1*errorlast-theta2*errorlastlast+noise				

Figure B-14 EXCEL, ARMA(2,2) Fit to Log-Linear Data

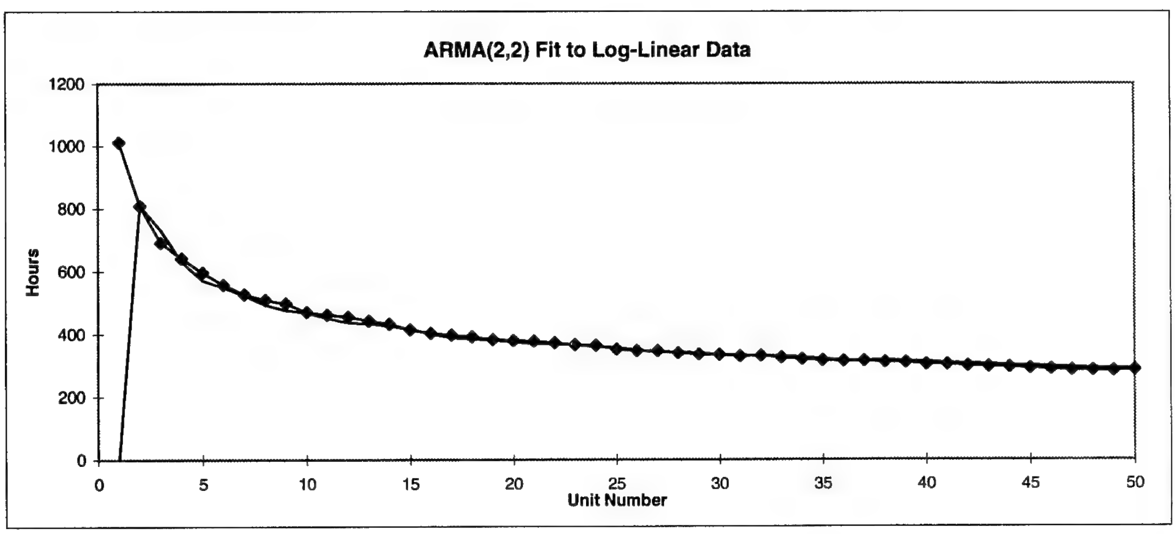


Table B-3 EXCEL, Simulated Log-Linear Data Base

The Simulated Log-Linear Data Base

Unit Number	Log-Lin(stoch)	Unit Number	Log-Lin(stoch)
1	1010.832269	26	347.6588376
2	808.3325458	27	346.8230994
3	691.2839896	28	340.5004826
4	641.1901161	29	336.3884482
5	595.4460444	30	335.0655174
6	557.2059351	31	329.6040636
7	527.0011364	32	332.1842007
8	508.4621019	33	325.4725805
9	496.9922034	34	320.8691874
10	469.848955	35	316.8387424
11	462.2715407	36	315.0144544
12	454.8076222	37	316.1912111
13	442.1848743	38	311.1881426
14	432.8762252	39	311.0490991
15	414.1127291	40	304.7384949
16	403.5124097	41	304.5376237
17	396.6960025	42	300.691177
18	391.530033	43	297.5358953
19	383.1869894	44	296.2756757
20	379.2670047	45	292.0449126
21	377.8492397	46	289.0213147
22	372.9927929	47	286.4419215
23	366.6170179	48	285.358444
24	364.765332	49	284.2516737
25	352.3002239	50	287.7423919

Appendix C

Fitting the Historical F-102 Data Using 500 Observations
(Work done in EXCEL)

Table C-1 Detailed Summary of Fitting the Historical F-102 Data Base

Fitting the F-102 Data					
Forsythe					
Parameters:	**a***	**b***	**cmin**		
	838249.88	-0.5144	0		
SSE:	5.503E+10				
Equation:	**Y(x)=a*x^b+cmin**				
Stanford-B					
Parameters:	**a***	**beta***	**n***		
	45456836.6	29.5607	-1.4187		
SSE:	4.540E+10				
Equation:	**Y(x)=a*(x+beta)^n**				
AR(1)					
Parameters:	**phi***	**const***			
	0.91522	4629.393			
SSE:	2.722E+10				
Equation:	**AR(1)= phi*(AR1-1)+const+noise**				
AR(2)					
Parameters:	**phi_1***	**phi_2***	**const***		
	0.9017230	-0.0214	11982.597		
SSE:	4.930E+09				
Equation:	**AR(2)= phi1*(AR2-1)+phi2*(AR2-2)+const+noise**				
AR(3)					
Parameters:	**phi_1***	**phi_2***	**phi_3***	**const***	
	0.84346	0.08390	-0.00332	4094.4724	
SSE:	2.637E+10				
Equation:	**AR(3)= phi1*(AR3-1)+phi2*(AR3-2)+phi3*(AR3-3)+const+noise**				
AR(4)					
Parameters:	**phi_1***	**phi_2***	**phi_3***	**phi_4***	**const***
	0.89845	-0.00194	-0.02330	-0.28369	12934.3652
SSE:	4.877E+09				
Equation:	**AR(4)= phi1*(AR4-1)+phi2*(AR4-2)+phi3*(AR4-3)+phi4*(AR4-4)+const+noise**				
ARMA(1,1)					
Parameters:	**muprime***	**theta***	**phi***		
	3426.4751	0.52344	0.93489		
SSE:	2.089E+10				
Equation:	**ARMA(1,1)=phi*(ARMA(1,1)-1)+muprime-theta*errorlast+noise**				

(more) Table C-1 Detailed Summary of Fitting the Historical F-102 Data Base

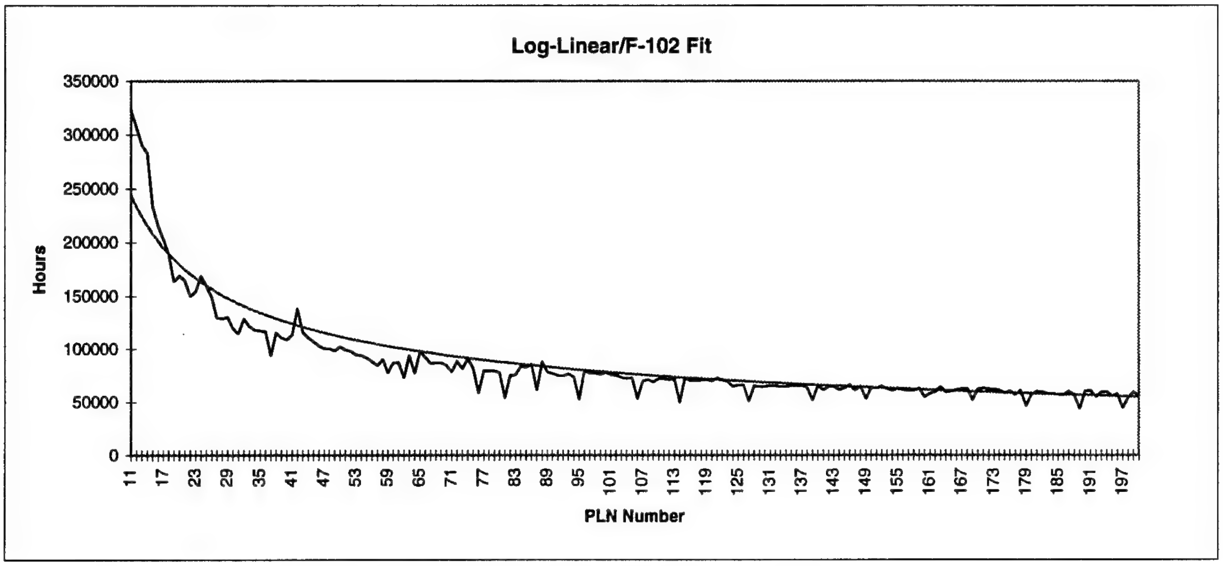
ARMA(1,2)					
Parameters:	muprime*	phi1*	theta1*	theta2*	
	3468.5690	0.93417	0.56678	-0.06678	
SSE:	2.079E+10				
Equation:	ARMA(1,2)=phi1*(ARMA(1,2)-1)+muprime-theta1*errorlast-theta2*errorlastlast+noise				
ARMA(2,1)					
Parameters:	muprime*	phi1*	phi2*	theta*	
	3400.5396	0.92509	0.01020	0.52183	
SSE:	2.087E+10				
Equation:	ARMA(2,1)=phi1*(ARMA(2,1)-1)+phi2*(ARMA(2,1)-2)+muprime-theta*errorlast+noise				
ARMA(2,2)					
Parameters:	muprime*	phi1*	phi2*	theta1*	theta2*
	3477.1898	0.93663	-0.00260	0.56949	-0.07003
SSE:	2.079E+10				
Equation:	ARMA(2,2)=phi1*(ARMA(2,2)-1)+phi2*(ARMA(2,2)-2)+muprime-theta1*errorlast-theta2*errorlastlast+noise				

Table C-2 Brief Summary of Fitting the Historical F-102 Data Base

Model	SSE	Rank
Log-Linear	5.503E+10	10
Forsythe	5.503E+10	10
Stanford-B	4.540E+10	9
AR(1)	2.722E+10	8
AR(2)	4.930E+09	2
AR(3)	2.637E+10	7
AR(4)	4.877E+09	1
ARMA(1,1)	2.089E+10	6
ARMA(1,2)	2.079E+10	4
ARMA(2,1)	2.087E+10	5
ARMA(2,2)	2.079E+10	3

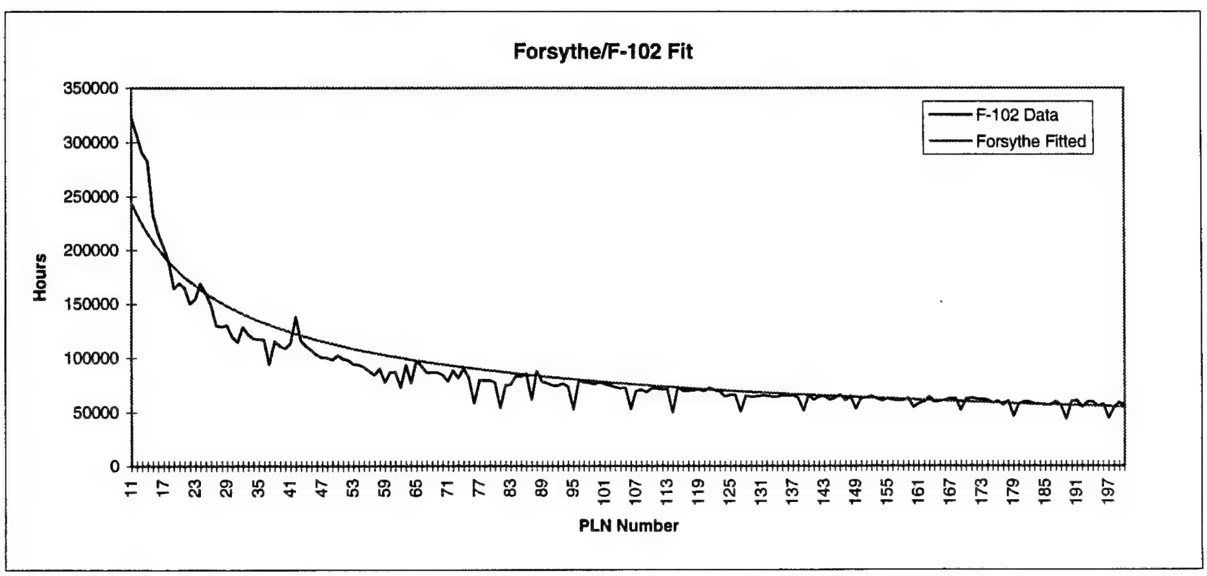
Parameters:	a*	b*
	838249.883	-0.514392071
SSE:	5.503E+10	
Equation:	Y(x)=a*x^b	

Figure C-1 EXCEL, Log-Linear Fit to F-102 Historical Data



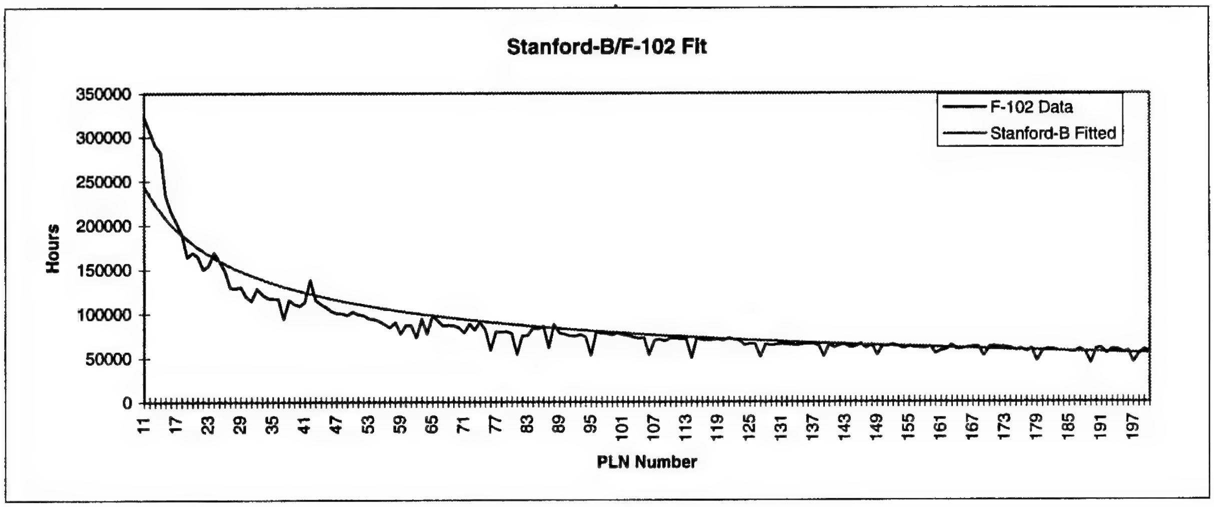
Parameters:	a*	b*	cmin*
	838249.883	-0.514392071	0
SSE:	5.503E+10		
Equation:	Y(x)=a*x^b+cmin		

Figure C-2 EXCEL, Forsythe Fit to F-102 Historical Data



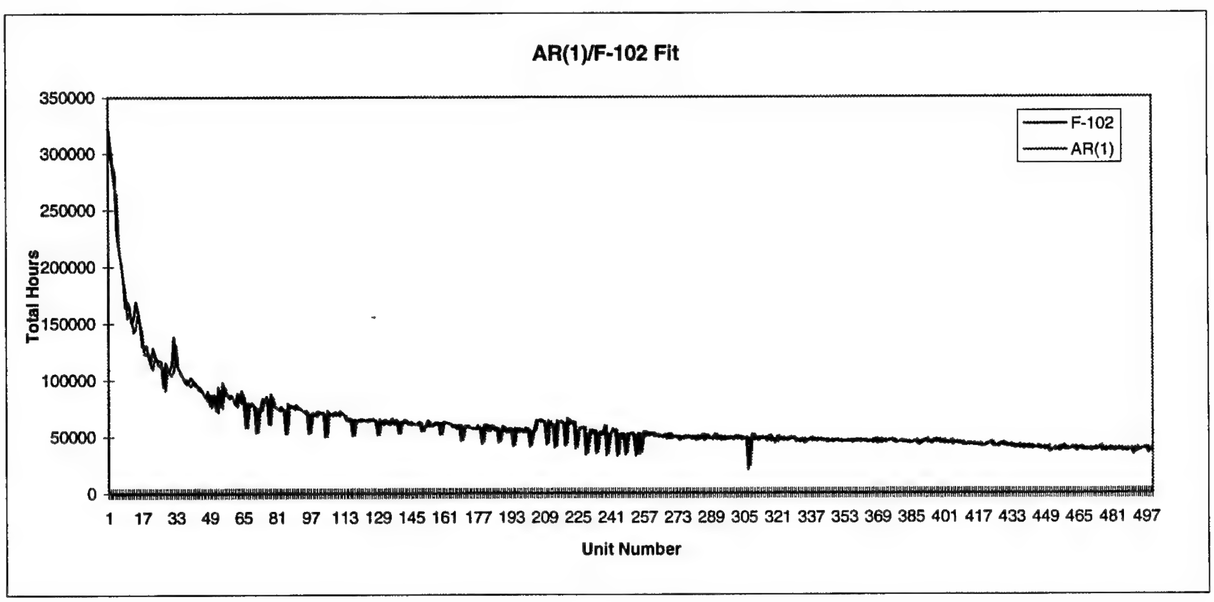
Parameters:	a*	beta*	n*
	45456836.63	29.56069417	-1.418710319
SSE:	4.540E+10		
Equation:	Y(x)=a*(x+beta)^n		

Figure C-3 EXCEL, Stanford-B Fit to F-102 Historical Data



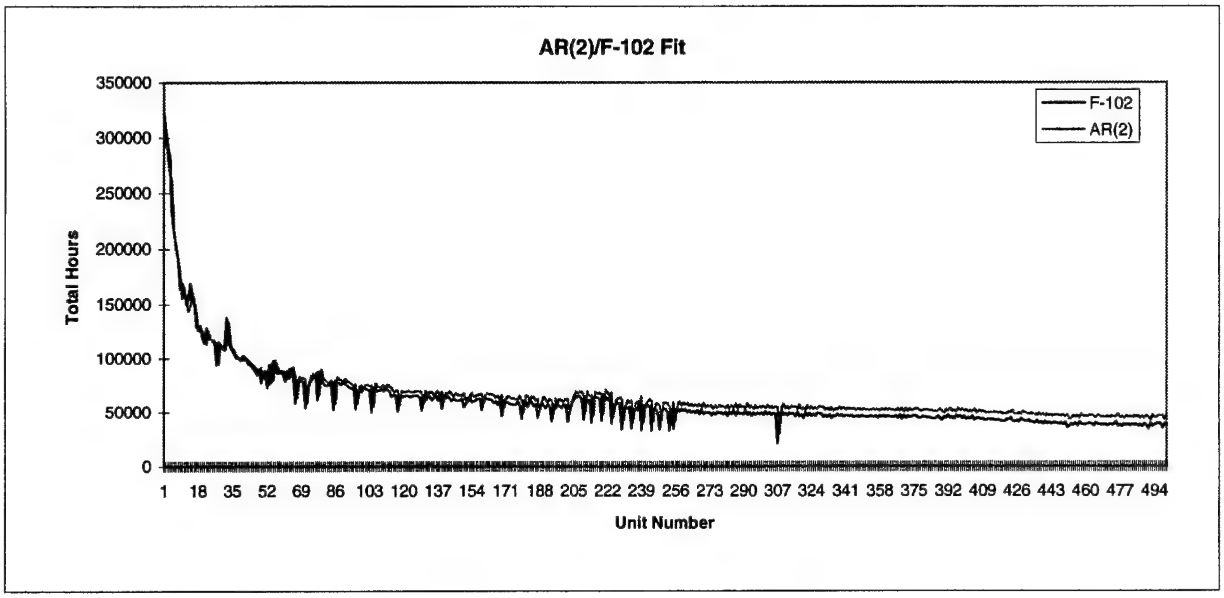
Parameters:	phi*	const*
	0.915219997	4629.392506
SSE:	2.722E+10	
Equation:	AR1(x)= phi*(Ysubx-1)+const+error	

Figure C-4 EXCEL, AR(1) Fit to F-102 Historical Data



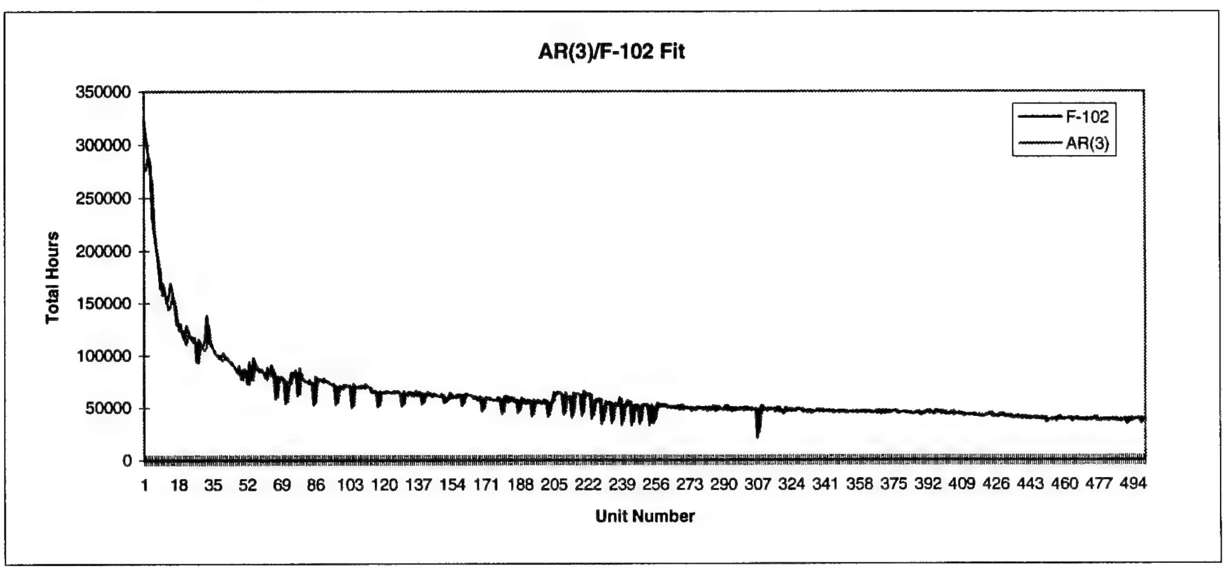
Parameters:	phi_1*	phi_2*	const*
	0.87589	-0.00905	17315.32351
SSE:	2.787E+10		
Equation:	AR2 = phi_1*(Ysubx-1)+ phi_2*(Ysubx-2)+const+error		

Figure C-5 EXCEL, AR(2) Fit to F-102 Historical Data



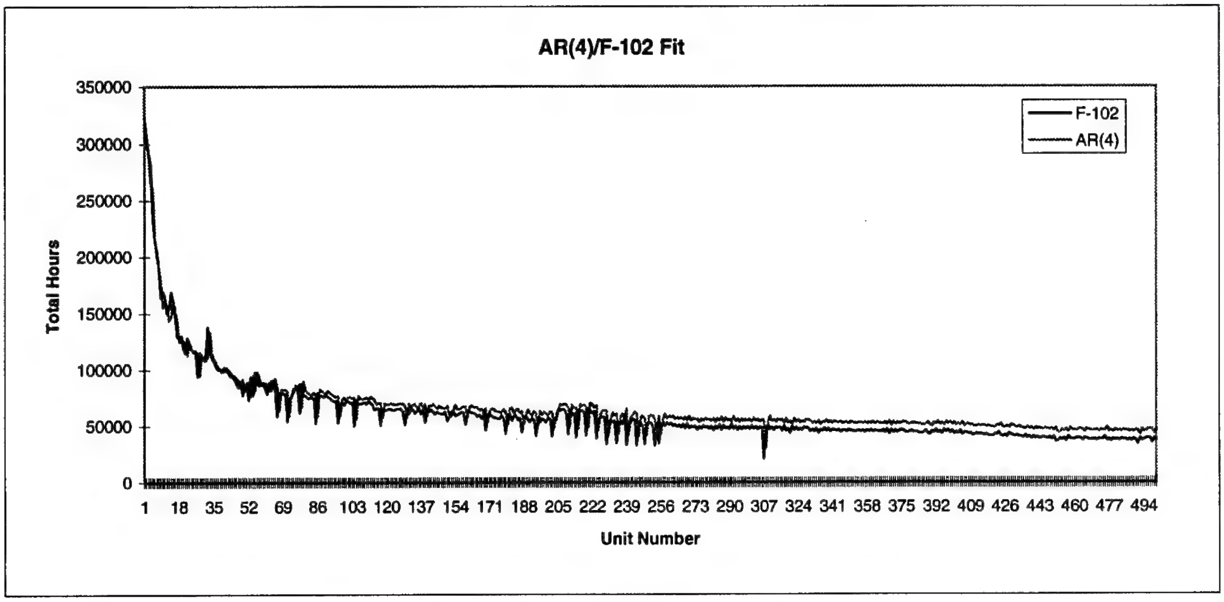
Parameters:	phi_1*	phi_2*	phi_3*	const*
	0.877114341	-0.095597136	0.160885068	3096.15527
SSE:	4.035E+10			
Equation:	AR3 = phi_1*(Ysubx-1)+ phi_2*(Ysubx-2)+phi_3*(Ysubx-2)+const+error			

Figure C-6 EXCEL, AR(3) Fit to F-102 Historical Data



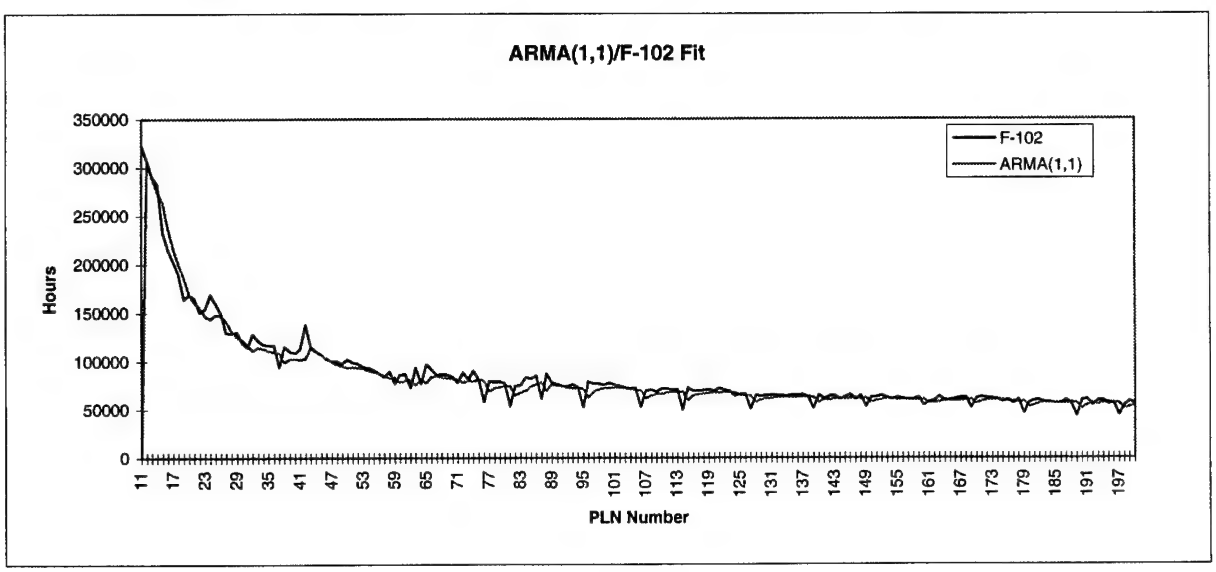
Parameters:	phi_1*	phi_2*	phi_3*	phi_4*	const*
	0.894643206	-0.138970848	0.159511384	-0.2836932	9123.75263
SSE:	2.349E+10				
Equation:	AR4 = phi_1*(Ysubx-1)+ phi_2*(Ysubx-2)+phi_3*(Ysubx-2)+const+error				

Figure C-7 EXCEL, AR(4) Fit to F-102 Historical Data



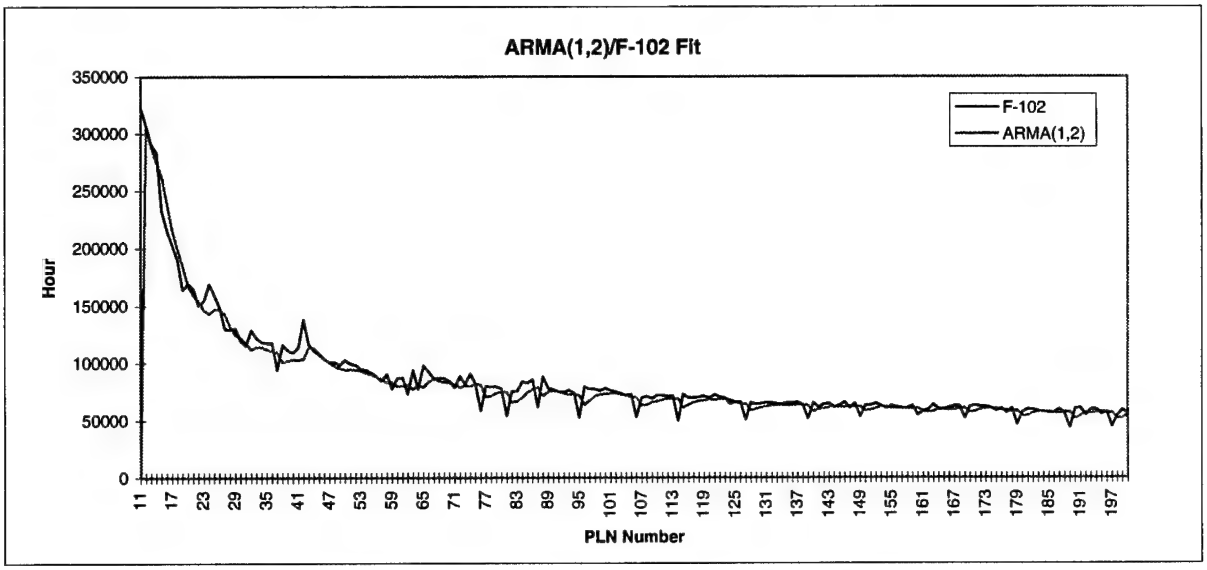
Parameters:	muprime*	theta*	phi*
	3426.4751	0.5234	0.9349
SSE:	2.089E+10		
Equation:	ARMA(1,1)=phi*(ARMA(1,1)-1)+muprime-theta*errorlast+noise		

Figure C-8 EXCEL, ARMA(1,1) Fit to F-102 Historical Data



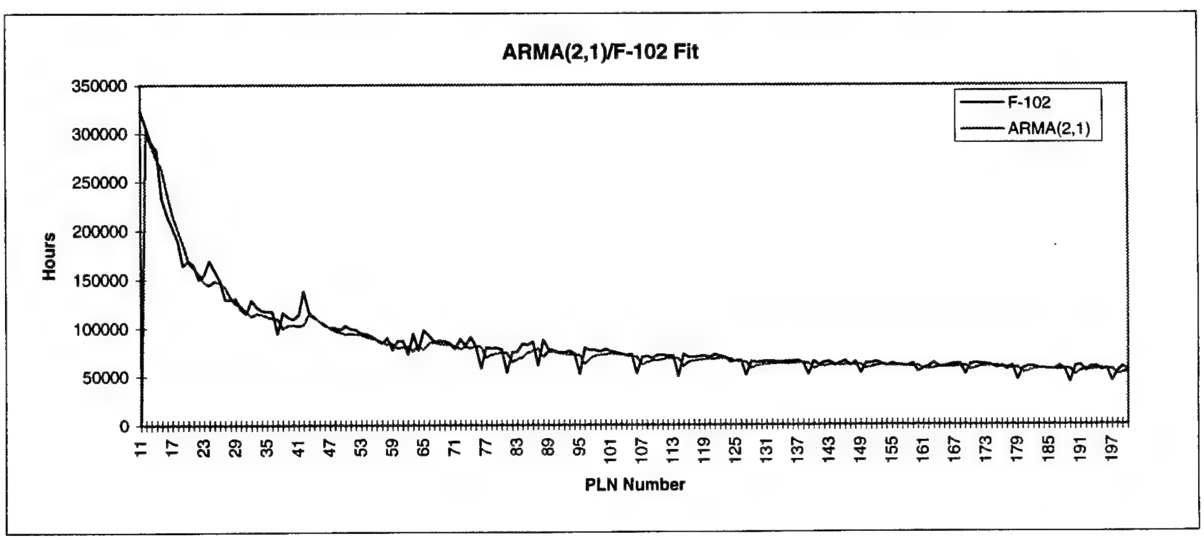
Parameters:	muprime*	phi1*	theta1*	theta2*
	3468.5690	0.9342	0.5668	-0.06678403
SSE:	2.079E+10			
Equation:	ARMA(1,2)=phi1*(ARMA(1,2)-1)+muprime-theta1*errorlast-theta2*errorlastlast+noise			

Figure C-9 EXCEL, ARMA(1,2) Fit to F-102 Historical Data



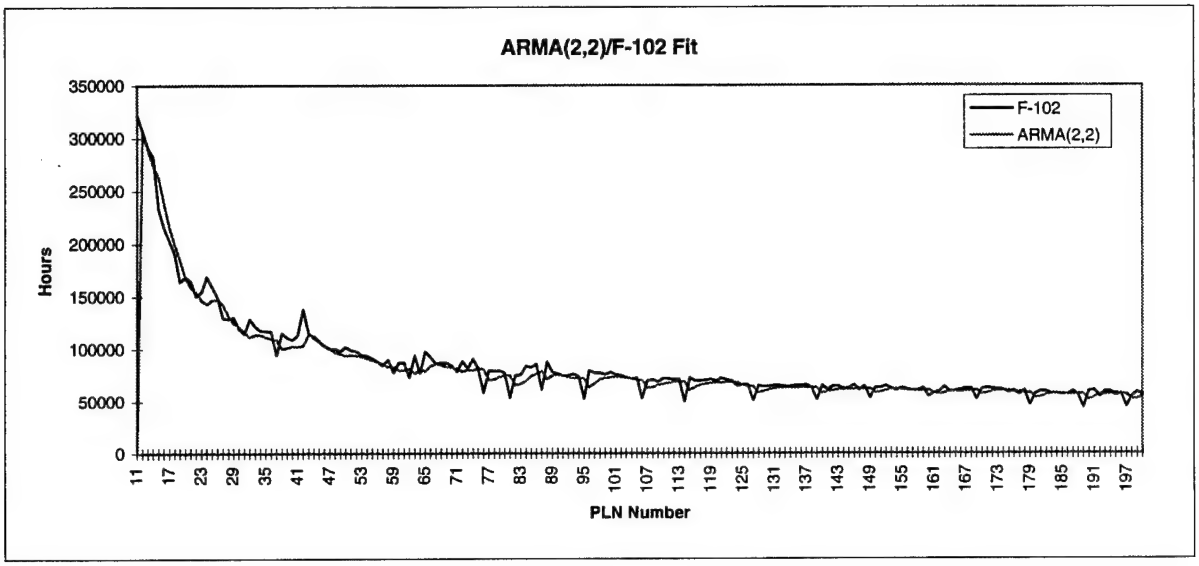
Parameters:	muprime*	phi1*	phi2*	theta*
	3400.5396	0.9251	0.010200353	0.5218
SSE:	2.087E+10			
Equation:	ARMA(2,1)=phi1*(ARMA(2,1)-1)+phi2*(ARMA(2,1)-2)+muprime-theta*errorlast+noise			

Figure C-10 EXCEL, ARMA(2,1) Fit to F-102 Historical Data



Parameters:	muprime*	phi1*	phi2*	theta1*	theta2*
	3477.1898	0.9366	-0.0026	0.569487058	-0.070032389
SSE:	2.079E+10				
Equation:	ARMA(2,2)=phi1*(ARMA(2,2)-1)+phi2*(ARMA(2,2)-2)+ muprime-theta1*errorlast-theta2*errorlastlast+noise				

Figure C-11 EXCEL, ARMA(2,2) Fit to F-102 Historical Data



Appendix D

Forecasting the Historical F-102 Data Using 20 Observations and a
Hold-out Sample of 480 Observations
(Work done in EXCEL)

Table D-1 Detailed Summary of Forecasting the Historical F-102 Data Base

Forecasting the F-102 Data Based on a 20 Unit History					
[illegible]					
Forsythe					
Parameters:	**a***	**b***	**cmin**		
	3772289.66	-1.0169	0		
SSE (1st 20):	2.965E+09				
SSE (all 500):	6.204E+11				
Equation:	**Y(x)=a*x^b+cmin**				
Stanford-B					
Parameters:	**a***	**beta***	**n***		
	1753527.5	-3.3272	-0.8136		
SSE (1st 20):	2.814E+09				
SSE (all 500):	1.560E+10				
Equation:	**Y(x)=a*(x+beta)^n**				
AR(1)					
Parameters:	**phi***	**const***			
	0.88739	11333.898			
SSE (1st 20):	2.452E+09				
SSE (all 500):	1.075E+12				
Equation:	**AR(1)= phi*(AR1-1)+const+noise**				
AR(2)					
Parameters:	**phi_1***	**phi_2***	**const***		
	0.9032423	-0.0341	14658.230		
SSE (1st 20):	2.350E+09				
SSE (all 500):	9.413E+09				
Equation:	**AR(2)= phi1*(AR2-1)+phi2*(AR2-2)+const+noise**				
AR(3)					
Parameters:	**phi_1***	**phi_2***	**phi_3***	**const***	
	0.89068	-0.00943	-0.03295	18459.07235	
SSE (1st 20):	2.257E+09				
SSE (all 500):	2.160E+12				
Equation:	**AR(3)= phi1*(AR3-1)+phi2*(AR3-2)+phi3*(AR3-3)+ const+noise**				
AR(4)					
Parameters:	**phi_1***	**phi_2***	**phi_3***	**phi_4***	**const***
	0.85381	-0.01134	0.02524	-0.08194	29783.44523
SSE (1st 20):	1.707E+09				
SSE (all 500):	4.063E+10				
Equation:	**AR(4)= phi1*(AR4-1)+phi2*(AR4-2)+phi3*(AR4-3)+ phi4*(AR4-4)+const+noise**				

(more) Table D-1 Detailed Summary of Forecasting the Historical F-102 Data Base

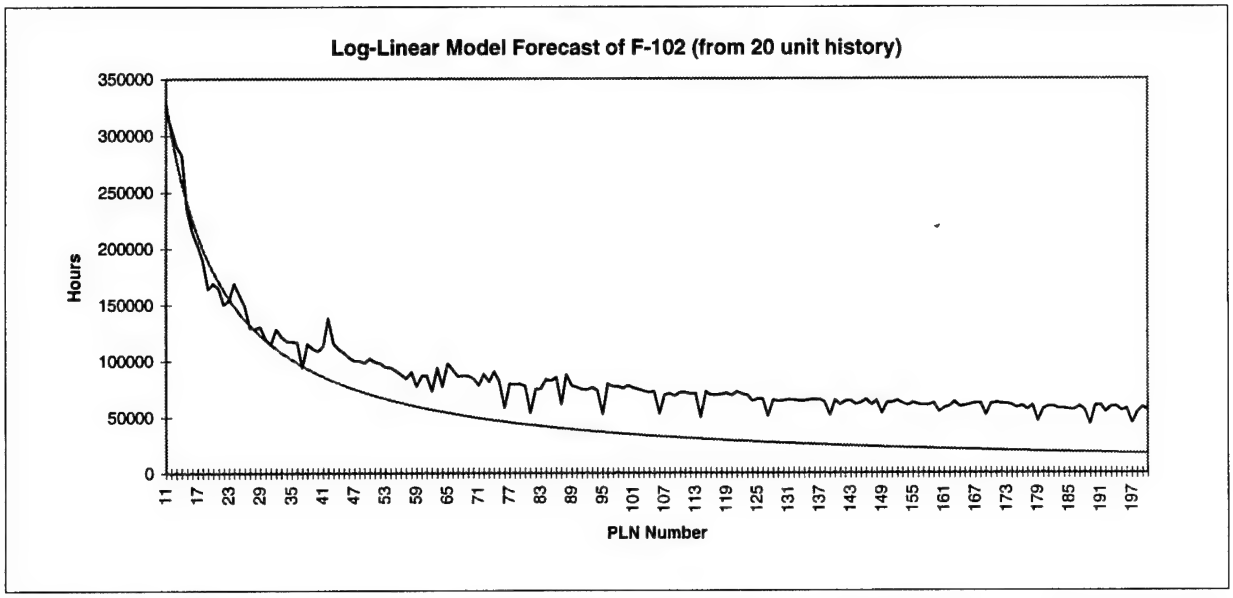
ARMA(1,1)					
Parameters:	**muprime***	**theta***	**phi***		
	11141.4558	0.17746	0.88825		
SSE (1st 20):	2.414E+09				
SSE (all 500):	1.036E+12				
Equation:	**ARMA(1,1)=phi*(ARMA(1,1)-1)+muprime-theta*errorlast+noise**				
ARMA(1,2)					
Parameters:	**muprime***	**phi1***	**theta1***	**theta2***	
	10202.6021	0.90340	1.04307	0.67739	
SSE (1st 20):	1.153E+09				
SSE (all 500):	1.294E+12				
Equation:	**ARMA(1,2)=phi1*(ARMA(1,2)-1)+muprime-theta1*errorlast-theta2*errorlastlast+noise**				
ARMA(2,1)					
Parameters:	**muprime***	**phi1***	**phi2***	**theta***	
	26833.4934	0.91517	-0.10895	1.72503	
SSE (1st 20):	8.848E+08				
SSE (all 500):	3.320E+12				
Equation:	**ARMA(2,1)=phi1*(ARMA(2,1)-1)+phi2*(ARMA(2,1)-2)+muprime-theta*errorlast+noise**				
ARMA(2,2)					
Parameters:	**muprime***	**phi1***	**phi2***	**theta1***	**theta2***
	15389.0325	0.93634	-0.07749	1.29884	0.46942
SSE (1st 20):	8.793E+08				
SSE (all 500):	1.458E+12				
Equation:	**ARMA(2,2)=phi1*(ARMA(2,2)-1)+phi2*(ARMA(2,2)-2)+muprime-theta1*errorlast- theta2*errorlastlast+noise**				

Table D-2 Brief Summary of Forecasting the Historical F-102 Data Base

Model	SSE (1st 20)	SSE (last 480)	SSE (all 500)	Rank (1st 20)	Rank (last 480)
Log-Linear	2.965E+09	6.175E+11	6.204E+11	10	4
Forsythe	2.965E+09	6.175E+11	6.204E+11	10	4
Stanford-B	2.814E+09	1.278E+10	1.560E+10	9	2
AR(1)	2.452E+09	1.073E+12	1.075E+12	8	6
AR(2)	2.350E+09	7.064E+09	9.413E+09	6	1
AR(3)	2.257E+09	2.158E+12	2.160E+12	5	9
AR(4)	1.707E+09	3.892E+10	4.063E+10	4	3
ARMA(1,1)	2.414E+09	1.034E+12	1.036E+12	7	5
ARMA(1,2)	1.153E+09	1.293E+12	1.294E+12	3	7
ARMA(2,1)	8.848E+08	3.319E+12	3.320E+12	2	10
ARMA(2,2)	8.793E+08	1.457E+12	1.458E+12	1	8

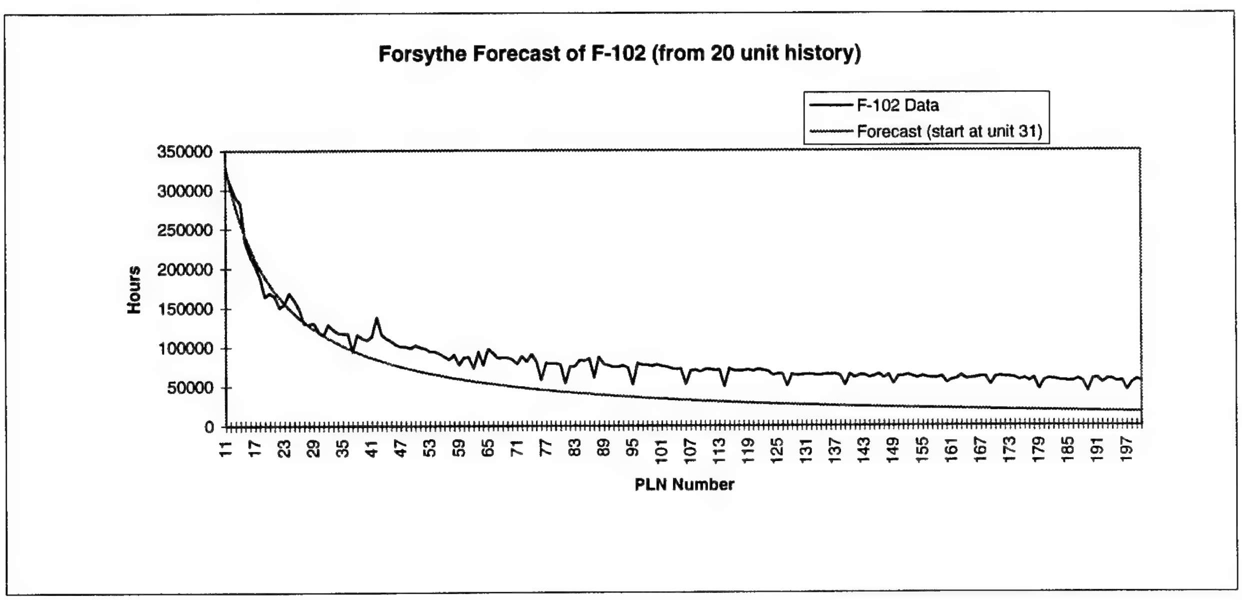
Parameters:	a*	b*
	3772289.657	-1.016921814
SSE(1st 20):	2.965E+09	
SSE(all 500):	6.204E+11	
Equation:	Y(x)=a*x^b	

Figure D-1 EXCEL, Log-Linear Forecast of F-102 Historical Data



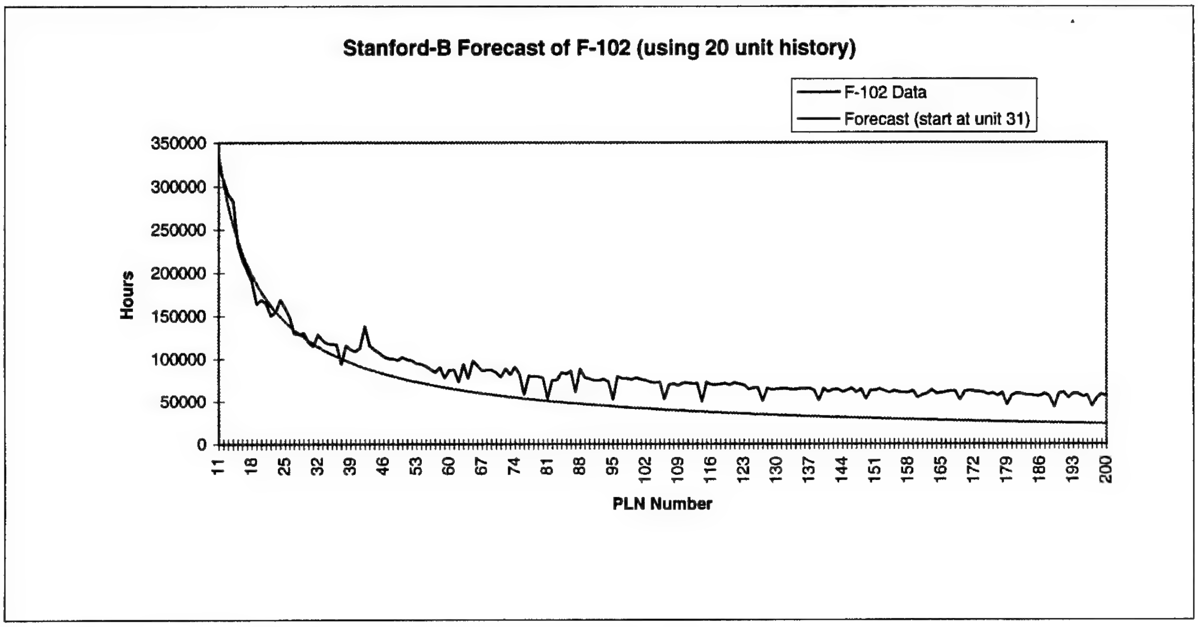
Parameters:	a*	b*	cmin*
	3772289.657	-1.016921814	0
SSE(1st 20):	2.965E+09		
SSE(all 500):	6.204E+11		
Equation:	Y(x)=a*x^b+cmin		

Figure D-2 EXCEL, Forsythe Forecast of F-102 Historical Data



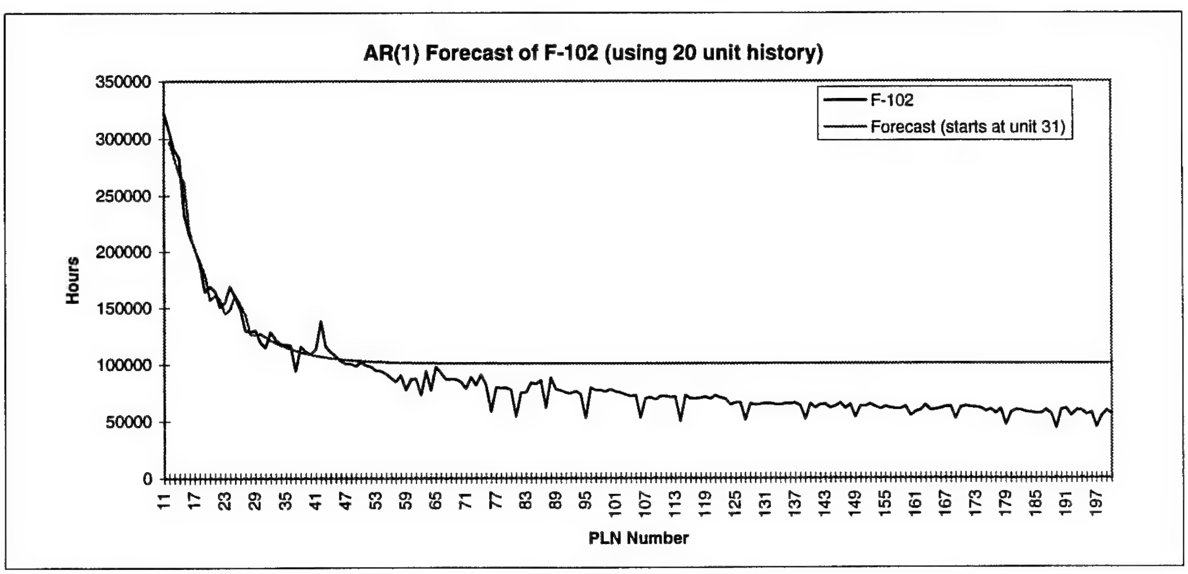
Parameters:	a*	beta*	n*
	1753527.473	-3.3272394	-0.813566665
SSE(1st 20):	2.814E+09		
SSE(all 500):	1.560E+10		
Equation:	Y(x)=a*(x+beta)^n		

Figure D-3 EXCEL, Stanford-B Forecast of F-102 Historical Data



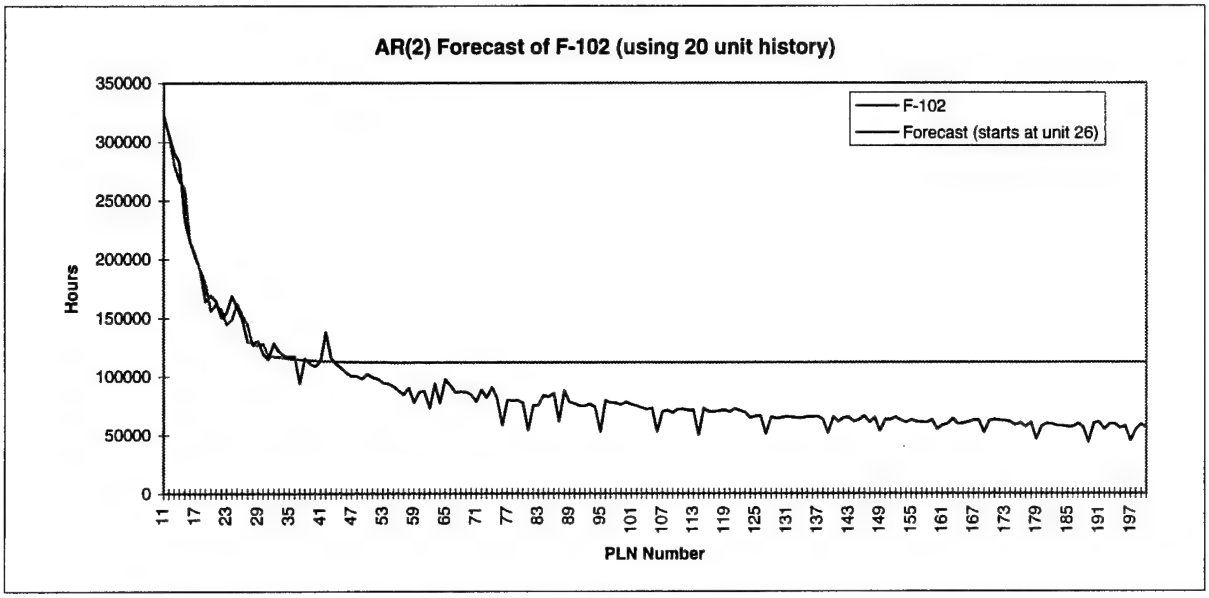
Parameters:	phi*	const*
	0.887387686	11333.89809
SSE (1st 20):	2.452E+09	
SSE (all 500):	1.075E+12	
Equation:	AR1 = phi*(Ysubx-1)+const+error	

Figure D-4 EXCEL, AR(1) Forecast of F-102 Historical Data



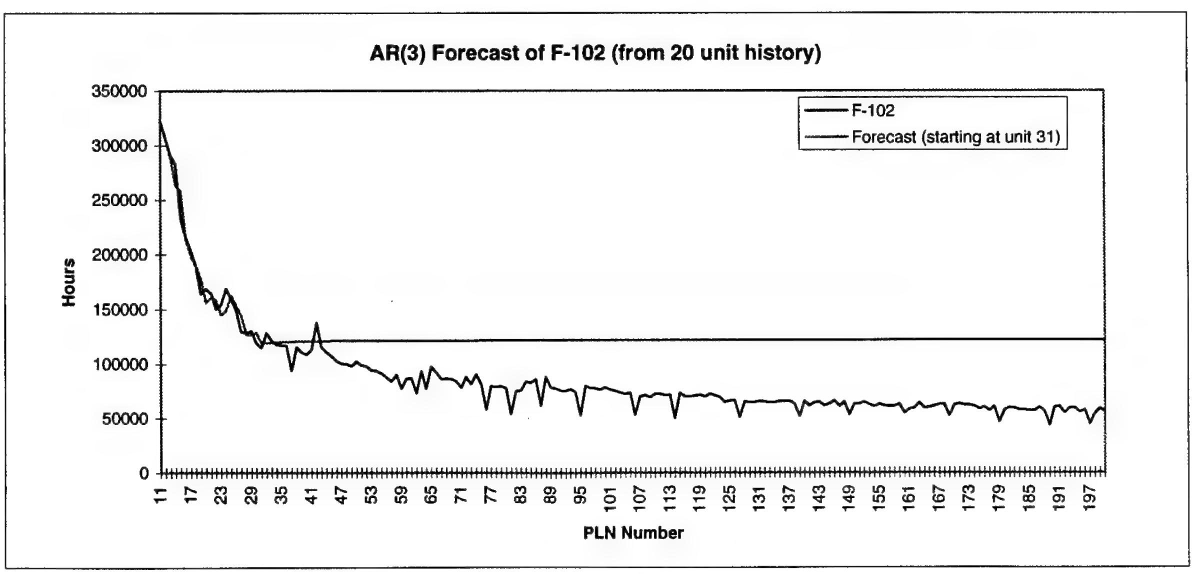
Parameters:	phi_1*	phi_2*	const*
	0.903242252	-0.03405886	14658.23013
SSE (1st 20):	2.350E+09		
SSE (all 500):	9.413E+09		
Equation:	AR(2) = phi_1*(Ysubx-1)+ phi_2*(Ysubx-2)+const+error		

Figure D-5 EXCEL, AR(2) Forecast of F-102 Historical Data



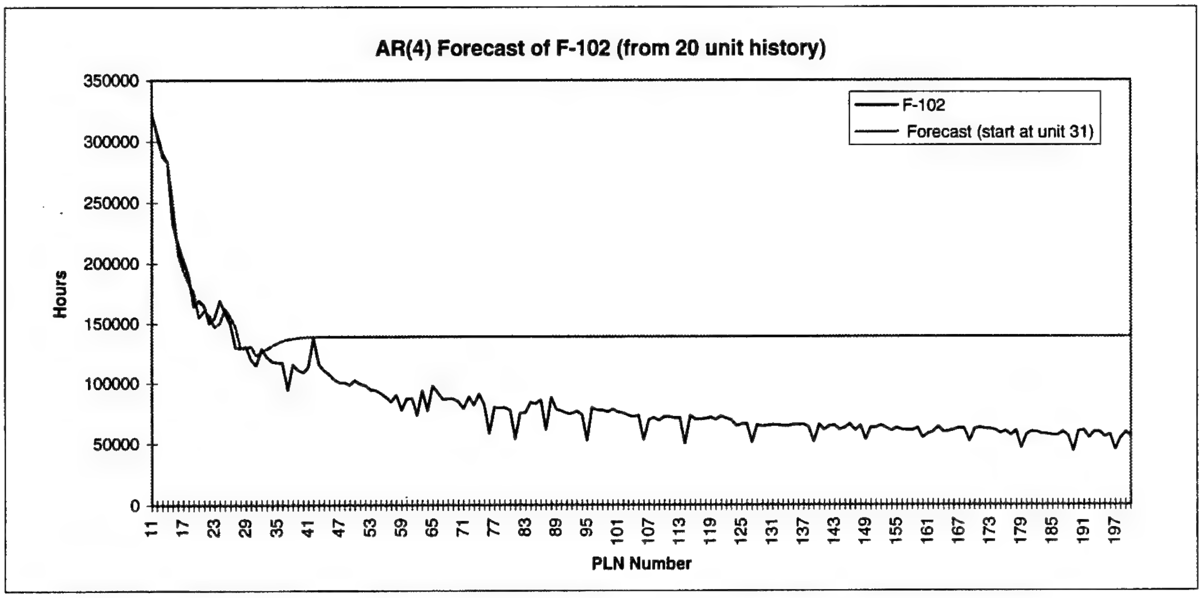
Parameters:	phi_1*	phi_2*	phi_3*	const*
	0.890677547	-0.009425592	-0.032951483	18459.07235
SSE (1st 20):	2.257E+09			
SSE (all 500):	2.160E+12			
Equation:	AR(3) = phi_1*(Ysubx-1)+ phi_2*(Ysubx-2)+phi_3*(Ysubx-2)+const+error			

Figure D-6 EXCEL, AR(3) Forecast of F-102 Historical Data



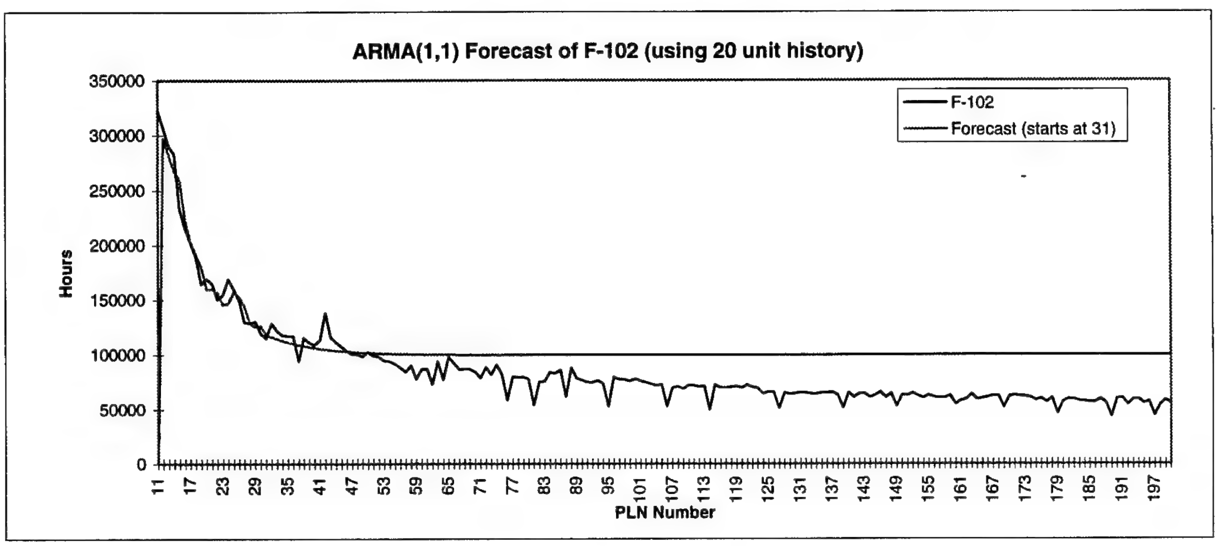
Parameters:	phi_1*	phi_2*	phi_3*	phi_4*	const*
	0.853808835	-0.011344383	0.025242833	-0.081935584	29783.44523
SSE (1st 20):	1.707E+09				
SSE (all 500):	4.063E+10				
Equation:	AR(4) = phi_1*(Ysubx-1)+ phi_2*(Ysubx-2)+phi_3*(Ysubx-2)+const+error				

Figure D-7 EXCEL, AR(4) Forecast of F-102 Historical Data



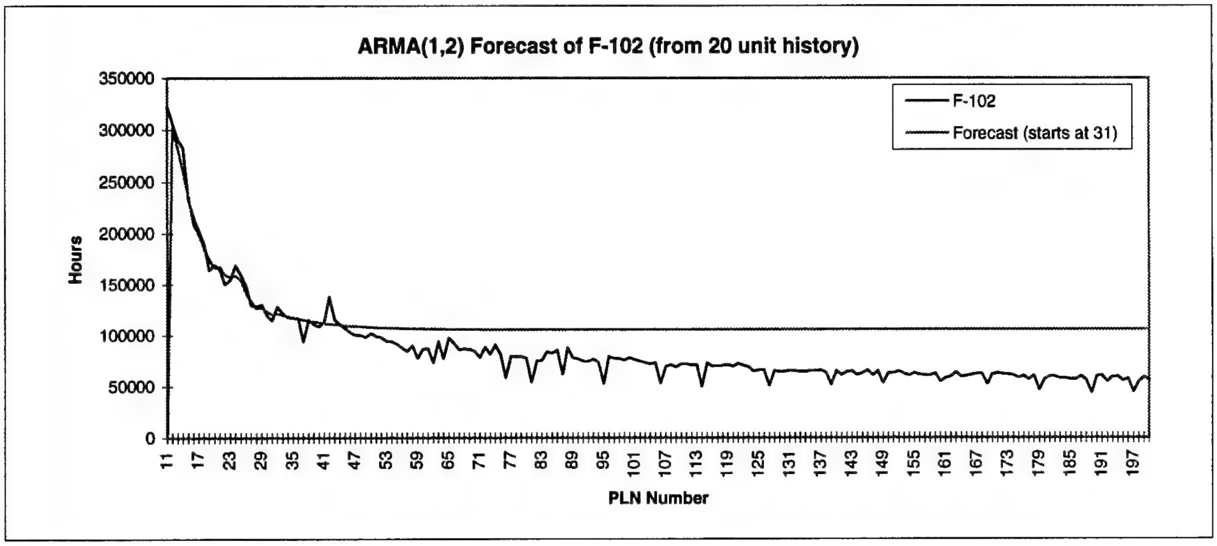
Parameters:	muprime*	theta*	phi*
	11141.4558	0.1775	0.8883
SSE (1st 25):	2.414E+09		
SSE (all 500):	1.036E+12		
Equation:	ARMA(1,1) = phi*(ARMA(1,1)-1)+muprime-theta*errorlast+noise		

Figure D-8 EXCEL, AR(1,1) Forecast of F-102 Historical Data



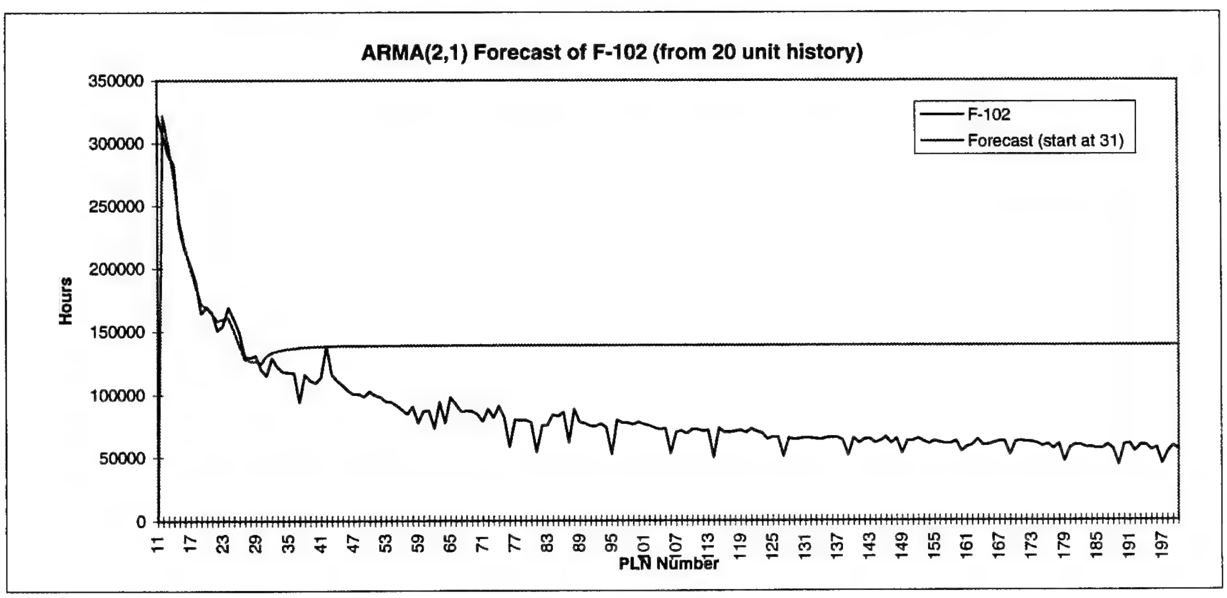
Parameters:	muprime*	phi1*	theta1*	theta2*
	10202.6021	0.9034	1.0431	0.677387668
SSE (1st 25):	1.153E+09			
SSE (all 500):	1.294E+12			
Equation:	ARMA(1,2) = phi1*(ARMA(1,2)-1)+muprime-theta1*errorlast-theta2*errorlastlast+noise			

Figure D-9 EXCEL, AR(1,2) Forecast of F-102 Historical Data



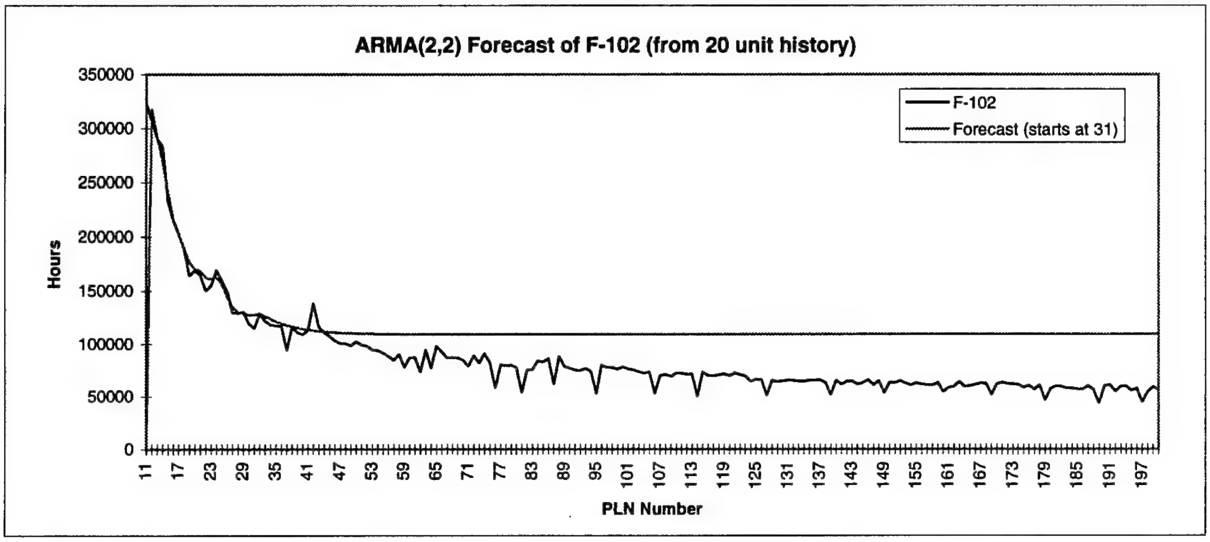
Parameters:	muprime*	phi1*	phi2*	theta*
	26833.4934	0.9152	-0.108951066	1.7250
SSE (1st 25):	8.848E+08			
SSE (all 500):	3.320E+12			
Equation:	ARMA(2,1) = phi1*(ARMA(2,1)-1)+phi2*(ARMA(2,1)-2)+muprime-theta*errorlast+noise			

Figure D-10 EXCEL, AR(2,1) Forecast of F-102 Historical Data



Parameters:	muprime*	phi1*	phi2*	theta1*	theta2*
	15389.0325	0.9363	-0.0775	1.298842272	0.469419732
SSE (1st 20):	8.793E+08				
SSE (all 500):	1.458E+12				
Equation:	ARMA(2,2) = phi1*(ARMA(2,2)-1)+phi2*(ARMA(2,2)-2)+ muprime-theta1*errorlast-theta2*errorlastlast+noise				

Figure D-11 EXCEL, AR(2,2) Forecast of F-102 Historical Data



Appendix E

The Historical F-102 Data Base - 500 Observations

Table E-1 The Historical F-102 Data

F-102 Data Base						
OBS	PLN	DelaySeq	TOTHRS	Lot	Contract#	DM
1	1	1	402475	1	5942	1
2	2	2	375849	1	5942	3
3	3	3	278963	2	5942	7
4	4	4	271223	2	5942	7
5	5	5	262498	2	5942	8
6	6	6	258078	2	5942	9
7	7	7	243726	2	5942	10
8	8	8	232766	2	5942	10
9	9	9	220833	2	5942	11
10	10	10	218827	2	5942	12
11	11	11	322447	3	5942	15
12	12	12	306736	3	5942	16
13	13	13	290470	3	5942	17
14	14	14	282951	3	5942	18
15	15	15	233125	4	5942	21
16	16	16	215379	4	5942	22
17	17	17	203122	4	5942	22
18	18	21	189770	4	5942	24
19	19	18	164120	4	5942	23
20	20	19	169080	4	5942	23
21	22	20	150387	4	5942	23
22	23	22	154606	4	5942	24
23	24	23	168896	4	5942	27
24	25	27	159485	4	5942	27
25	26	25	149109	4	5942	27
26	27	32	129792	5	5942	28
27	28	24	128958	5	5942	27
28	29	31	130389	5	5942	28
29	31	26	114872	4	23903	26
30	32	36	128470	5	5942	28
31	33	35	121932	5	5942	28
32	34	29	117969	5	5942	28
33	35	33	117364	5	5942	28
34	36	37	116895	5	5942	29
35	37	30	94174	5	23903	27
36	38	40	115466	5	5942	29
37	39	28	111137	5	5942	27
38	40	47	108890	5	5942	30
39	41	49	113487	5	5942	30
40	43	55	115846	5	5942	30
41	44	39	111029	5	5942	29
42	45	44	107397	5	5942	29
43	21	38	164751	4	5942	29
44	30	46	119597	5	5942	29
45	47	52	100520	6	23903	31
46	48	50	100504	6	23903	31

Table E-1 The Historical F-102 Data

F-102 Data Base

OBS	PLN	DelaySeq	TOTHRS	Lot	Contract#	DM
47	49	34	98316	6	23903	30
48	51	57	99317	6	23903	32
49	52	41	98162	6	23903	30
50	53	42	94614	6	23903	30
51	55	43	91628	6	23903	30
52	56	45	88098	6	23903	30
53	58	48	90164	6	23903	30
54	60	51	86840	6	23903	31
55	61	53	87197	6	23903	31
56	63	54	93812	6	23903	32
57	65	56	97725	6	23903	32
58	66	59	92401	7	23903	33
59	67	58	86481	7	23903	33
60	69	60	86830	7	23903	33
61	70	61	84477	7	23903	33
62	72	70	88589	7	23903	34
63	62	75	73200	9	23903	34
64	73	62	81738	7	23903	33
65	74	109	90762	7	23903	35
66	75	64	81865	7	23903	34
67	77	65	79839	7	23903	34
68	78	66	79358	7	23903	34
69	79	67	79651	7	23903	34
70	80	68	77606	7	23903	34
71	82	69	75097	7	23903	34
72	46	86	103137	6	23903	35
73	64	81	77297	9	23903	34
74	83	71	75510	7	23903	34
75	84	145	83796	7	23903	35
76	85	72	82662	7	23903	35
77	86	73	85787	7	23903	35
78	88	146	87891	7	23903	35
79	89	74	78264	7	23903	35
80	90	76	76967	7	23903	35
81	91	77	74963	7	23903	35
82	92	78	74628	7	23903	35
83	93	79	76608	7	23903	35
84	94	80	73689	7	23903	35
85	96	82	79580	8	29264	35
86	97	84	77425	8	29264	36
87	42	63	137990	6	23903	33
88	98	83	77409	8	29264	35
89	99	88	75854	8	29264	36
90	100	85	78187	8	29264	36
91	101	87	76044	8	29264	36
92	102	89	75017	8	29264	36

Table E-1 The Historical F-102 Data

F-102 Data Base

OBS	PLN	DelaySeq	TOTHRS	Lot	Contract#	DM
93	103	91	73337	8	29264	36
94	104	94	72091	8	29264	36
95	105	92	72916	8	29264	36
96	107	93	69718	8	29264	36
97	108	106	71085	8	29264	37
98	109	95	68818	8	29264	36
99	110	96	72137	8	29264	37
100	111	97	72101	8	29264	37
101	112	99	70917	8	29264	37
102	113	98	71333	8	29264	37
103	115	100	72966	8	29264	37
104	116	101	69869	8	29264	37
105	117	103	69895	8	29264	37
106	118	104	70448	8	29264	37
107	119	105	71490	8	29264	37
108	120	108	69560	8	29264	37
109	132	120	65290	9	29264	37
110	133	121	64415	9	29264	37
111	134	122	64269	9	29264	37
112	135	123	65396	9	29264	37
113	136	124	65398	9	29264	37
114	137	123	65747	9	29264	37
115	138	127	63262	9	29264	38
116	140	126	65332	9	29264	38
117	141	128	61274	9	29264	38
118	142	129	64319	9	29264	38
119	143	130	64793	9	29264	38
120	144	131	61270	9	29264	38
121	145	134	63046	9	29264	38
122	146	132	65928	9	29264	38
123	147	135	60844	9	29264	39
124	148	137	64753	9	29264	38
125	59	133	77679	9	23903	38
126	68	102	86933	9	23903	37
127	71	90	78669	9	23903	36
128	121	111	72530	9	29264	37
129	122	112	70642	9	29264	37
130	123	110	69385	9	29264	37
131	124	113	64465	9	29264	37
132	125	114	66052	9	29264	37
133	126	115	66023	9	29264	37
134	128	116	65284	9	29264	37
135	129	117	64232	9	29264	37
136	130	118	64695	9	29264	37
137	131	119	65503	9	29264	37
138	150	136	63054	9	29264	38

Table E-1 The Historical F-102 Data

F-102 Data Base

OBS	PLN	DelaySeq	TOTHRS	Lot	Contract#	DM
139	151	138	63048	9	29264	38
140	152	139	64929	9	29264	38
141	153	740	62522	9	29264	38
142	154	142	60538	9	29264	38
143	155	141	62861	9	29264	38
144	156	147	61370	9	29264	38
145	157	148	60747	9	29264	39
146	158	143	60665	9	29264	38
147	159	149	62824	9	29264	38
148	161	150	58087	9	29264	38
149	162	152	59473	9	29264	38
150	163	157	63951	9	29264	38
151	164	153	59320	9	29264	38
152	165	156	60055	9	29264	38
153	166	154	61220	9	29264	38
154	167	155	62458	9	29264	38
155	168	157	62412	9	29264	38
156	170	158	61843	9	29264	38
157	171	159	63077	9	29264	38
158	172	160	62071	9	29264	38
159	173	161	61858	10	29264	38
160	174	162	60979	10	29264	38
161	175	163	58349	10	29264	38
162	176	164	60204	10	29264	38
163	177	167	56691	10	29264	38
164	178	165	60527	10	29264	38
165	180	166	57210	10	29264	38
166	181	168	59645	10	29264	38
167	182	170	59433	10	29264	38
168	50	171	102354	6	23903	38
169	54	107	94057	8	23903	37
170	57	144	84495	8	23903	38
171	183	169	57653	10	29264	39
172	184	172	57566	10	29264	39
173	185	173	56691	10	29264	39
174	186	174	56621	10	29264	39
175	187	176	59791	10	29266	39
176	188	177	56079	10	29264	39
177	76	181	58506	10	23903	39
178	190	178	59994	10	29264	39
179	191	179	60757	10	29264	39
180	192	180	54645	10	29264	39
181	193	182	59312	10	29264	39
182	194	183	59483	10	29264	39
183	195	184	55581	10	29264	39
184	196	185	57455	10	29264	39

Table E-1 The Historical F-102 Data

F-102 Data Base

OBS	PLN	DelaySeq	TOTHRS	Lot	Contract#	DM
185	198	175	54241	10	29264	39
186	199	186	59042	10	29264	39
187	200	187	56001	10	29264	39
188	201	188	53381	10	29264	39
189	202	189	57139	10	29264	39
190	203	190	55403	10	29264	39
191	201	191	54271	10	29264	39
192	81	192	54031	10	23903	39
193	206	193	57332	10	29264	39
194	207	194	53560	10	29264	39
195	208	193	56839	10	29264	39
196	209	196	55439	10	29264	39
197	210	197	53354	10	29264	39
198	211	198	57169	10	29264	39
199	212	199	56625	10	29264	39
200	214	200	53698	10	29264	39
201	215	201	55117	10	29264	39
202	216	202	64410	11	31174	40
203	217	203	63457	11	31174	40
204	218	204	64855	11	31174	40
205	219	203	64709	11	31174	40
206	220	206	61493	11	31174	40
207	87	207	61633	11	23903	40
208	222	208	64154	11	31174	40
209	223	209	60020	11	31174	40
210	224	210	61057	11	31174	40
211	226	211	64077	11	31174	40
212	227	212	62750	11	31174	40
213	228	213	63673	11	31174	40
214	229	214	61986	11	31174	40
215	231	215	65932	11	31174	40
216	232	216	64059	11	31174	40
217	233	217	61146	11	31174	40
218	234	218	63329	11	31174	40
219	236	219	54223	12	31174	40
220	237	220	57406	12	31174	40
221	95	221	52714	11	23903	40
222	238	222	57999	12	23903	40
223	239	223	58051	12	31174	40
224	241	224	47604	12	31174	40
225	242	225	55653	12	31174	40
226	243	226	53132	12	31174	40
227	244	227	55019	12	31174	40
228	246	228	51793	12	31174	40
229	247	229	54301	12	31174	40
230	248	230	51429	12	31174	40

Table E-1 The Historical F-102 Data

F-102 Data Base

OBS	PLN	DelaySeq	TOTHRS	Lot	Contract#	DM
231	249	231	59606	12	31174	46
232	251	232	51161	12	31174	40
233	252	233	53072	12	31174	40
234	253	234	56653	12	31174	40
235	106	235	53057	11	23903	41
236	254	236	55143	12	31171	40
237	256	237	50899	12	31174	40
238	257	238	52395	12	31174	40
239	258	239	50183	12	31174	40
240	259	240	52217	12	31174	40
241	261	241	52656	12	31174	40
242	262	245	52818	12	31174	41
243	263	242	49485	12	31174	40
244	265	246	52517	12	31174	41
245	266	251	49671	12	31174	41
246	267	247	54124	12	31174	41
247	269	248	50204	12	31174	41
248	270	243	52963	12	31174	41
249	271	249	51224	12	31174	41
250	114	244	50005	11	23903	41
251	273	250	52793	12	31174	41
252	274	252	49813	12	31174	41
253	275	253	51312	12	31174	41
254	277	254	49304	12	31174	41
255	278	257	50879	12	31174	41
256	279	255	48798	12	31174	41
257	281	256	51624	12	31174	41
258	282	258	47738	12	31174	41
259	283	259	52435	12	31174	41
260	285	262	48038	12	31174	41
261	127	263	50935	12	23903	42
262	286	260	49978	12	31174	41
263	287	261	49255	12	31174	41
264	289	264	49755	12	31174	41
265	290	271	46508	12	31174	41
266	291	277	49555	12	31174	41
267	293	265	47469	12	31174	41
268	294	272	50337	12	31174	41
269	295	266	48865	12	31174	41
270	139	273	51644	12	23903	42
271	297	267	49463	12	31174	41
272	298	268	47582	12	31174	41
273	299	269	49975	12	31774	41
274	301	274	46285	12	31174	41
275	302	275	50195	12	31174	41
276	303	283	47662	12	31174	42

Table E-1 The Historical F-102 Data

F-102 Data Base

OBS	PLN	DelaySeq	TOTHRS	Lot	Contract#	DM
277	149	270	53218	13	29264	43
278	305	278	51616	12	31174	42
279	306	279	46355	12	31174	42
280	307	285	50820	12	31174	42
281	308	276	48181	12	31174	41
282	309	280	48972	12	31174	42
283	310	284	47284	12	31174	42
284	311	286	51997	12	31174	42
285	312	289	47163	12	31174	42
286	313	281	49759	12	31174	42
287	314	282	46257	72	31174	42
288	160	287	54706	13	29264	43
289	315	290	49545	12	37174	42
290	316	291	47045	72	31174	42
291	317	292	49222	12	31174	42
292	318	288	47705	12	31174	42
293	319	293	49419	12	31174	42
294	320	297	46921	12	31174	42
295	321	295	50453	12	31174	42
296	322	296	47644	12	31174	42
297	169	298	51951	13	29264	43
298	323	294	49567	12	31174	42
299	324	311	46926	13	31174	42
300	325	299	48883	13	31174	42
301	326	308	48317	13	31174	42
302	327	309	20921	13	31174	42
303	328	300	46097	13	31174	42
304	329	301	51757	13	31174	42
305	330	302	46997	13	31174	42
306	179	324	46463	14	29264	44
307	331	303	48965	13	31174	42
308	332	312	46431	13	31174	42
309	333	310	49284	13	31174	42
310	334	304	48369	13	31174	42
311	335	305	48940	13	31174	42
312	336	313	45578	13	31174	42
313	337	319	50382	13	37174	43
314	338	306	45772	13	31174	42
315	339	307	47439	13	31174	42
316	189	377	43656	14	29264	44
317	340	467	43759	13	31174	46
318	341	314	49828	13	31174	42
319	342	315	46317	13	31174	42
320	343	316	48883	13	31174	42
321	344	320	46107	13	31174	43
322	345	317	47944	13	31174	43

Table E-1 The Historical F-102 Data

F-102 Data Base						
OBS	PLN	DelaySeq	TOTHRS	Lot	Contract#	DM
323	346	321	46362	13	31174	43
324	347	378	49038	13	31174	43
325	348	325	48512	13	31174	43
326	349	322	48172	13	31174	43
327	350	323	45123	13	31174	43
328	197	365	44669	14	29264	44
329	351	326	46952	13	31174	43
330	352	327	45029	13	31174	43
331	353	331	47745	13	31174	43
332	354	328	43882	13	31174	43
333	355	329	46630	13	31174	43
334	356	330	46004	13	31174	43
335	357	332	48265	13	31174	43
336	358	333	45810	13	31174	43
337	359	336	47808	13	31174	43
338	360	348	45384	13	31174	43
339	205	384	41615	14	29264	45
340	361	334	47072	13	31174	3
341	362	337	46680	13	31174	43
342	363	335	45825	13	31174	43
343	364	338	46737	13	31174	43
344	365	349	44783	13	31174	43
345	366	339	46061	13	31174	43
346	367	340	46326	13	37174	43
347	368	347	44932	13	31174	43
348	369	341	46868	13	31174	43
349	370	342	45799	13	31774	43
350	213	385	40989	14	29264	45
351	371	343	45658	13	31174	43
352	372	353	44921	13	31174	43
353	373	350	45909	13	31174	43
354	374	344	45890	13	31174	43
355	375	345	45209	13	31174	43
356	376	346	46284	13	31174	43
357	377	351	44480	13	31174	43
358	378	354	46528	13	31174	43
359	379	355	45290	13	31174	43
360	380	352	46779	13	31174	43
361	220	366	43022	14	29264	44
362	381	356	45011	13	31174	43
363	382	357	46331	13	31174	43
364	383	358	45536	13	31174	43
365	384	397	45241	13	31174	44
366	385	359	44991	13	31174	43
367	386	360	46435	13	31174	43
368	387	361	44128	13	31174	43

Table E-1 The Historical F-102 Data

F-102 Data Base

OBS	PLN	DelaySeq	TOTHRS	Lot	Contract#	DM
369	388	367	47348	13	31174	44
370	389	362	43402	13	31174	43
371	390	368	47047	13	31174	44
372	391	369	44173	13	31174	44
373	225	410	40271	14	29264	46
374	392	363	46565	13	37174	44
375	393	370	44759	13	31174	44
376	394	364	45930	13	31174	44
377	395	378	46869	13	37174	44
378	396	373	46292	13	31174	44
379	397	380	44844	13	31174	44
380	398	371	44195	13	31174	44
381	399	372	45243	13	31174	44
382	400	386	45916	13	31174	44
383	401	375	44838	13	31174	44
384	402	381	44763	13	31174	44
385	230	421	41948	14	29264	45
386	403	374	44645	13	31174	44
387	404	376	43948	13	31174	44
388	405	389	43744	13	31174	44
389	406	382	44580	13	31174	44
390	407	391	46038	13	37174	44
391	408	387	43652	13	31174	44
392	409	379	42371	13	31174	44
393	410	383	45607	13	31174	44
394	411	388	44552	13	31174	44
395	412	392	47014	13	31174	44
396	413	393	44289	13	31174	44
397	414	394	45401	13	31174	44
398	235	428	39196	14	29264	46
399	415	390	43368	13	31174	44
400	416	403	46979	14	31174	44
401	417	398	44974	14	31174	44
402	418	404	46598	14	31174	44
403	419	395	43842	14	31174	44
404	420	399	46131	14	31174	44
405	421	396	43247	14	31174	44
406	422	400	44734	14	31174	44
407	423	401	43501	14	31174	44
408	424	402	46129	14	31174	44
409	425	405	42443	14	31174	44
410	240	454	34031	14	29264	46
411	426	417	44328	14	31174	45
412	427	406	44473	14	31174	44
413	428	407	43418	14	31174	44
414	429	408	43775	14	31174	44

Table E-1 The Historical F-102 Data

F-102 Data Base

OBS	PLN	DelaySeq	TOTHRS	Lot	Contract#	DM
415	430	409	42007	14	31174	44
416	431	411	42537	14	31174	44
417	432	422	43270	14	31174	45
418	433	418	42650	14	31174	45
419	434	412	43261	14	31174	45
420	435	413	42486	14	31174	45
421	436	414	42083	14	31174	45
422	437	419	42678	14	31174	45
423	245	440	34969	14	29264	46
424	438	425	41673	14	31174	45
425	439	415	41431	14	31174	45
426	440	420	41347	14	31174	45
427	441	416	43084	14	31174	45
428	442	429	42843	14	31174	45
429	443	462	44814	14	31174	45
430	444	441	41706	14	31174	45
431	445	423	41108	14	31174	45
432	496	426	41091	14	31174	45
433	447	424	42091	14	31174	45
434	448	430	40599	14	31174	45
435	250	470	33149	14	29261	46
436	449	437	43760	14	31174	45
437	450	427	42527	14	31174	45
438	451	438	41942	14	31174	45
439	452	431	41095	14	31174	45
440	453	432	42314	14	31174	43
441	454	466	40751	14	31174	43
442	455	434	40744	14	31174	45
443	456	439	39510	14	31174	45
444	457	433	41948	14	31174	45
445	458	442	39649	14	31174	45
446	459	435	41034	14	31174	45
447	460	436	38751	14	31174	45
448	255	475	32806	14	29264	46
449	461	443	41160	14	31174	45
450	462	444	39329	14	31174	45
451	463	445	40440	14	31174	15
452	464	446	38930	14	31174	45
453	465	447	39816	14	31174	45
454	466	455	40240	74	31174	45
455	467	448	39527	14	31174	45
456	468	449	38800	14	31174	45
457	469	450	40127	14	31174	45
458	470	456	38137	14	31174	45
459	471	451	40364	14	31174	45
460	472	559	35847	14	31174	48

Table E-1 The Historical F-102 Data

F-102 Data Base

OBS	PLN	DelaySeq	TOTHRS	Lot	Contract#	DM
461	265	499	34639	11	29264	47
462	473	452	36716	14	31174	45
463	474	457	38847	14	31174	45
464	475	458	37680	14	31170	45
465	476	459	39414	14	31174	45
466	477	453	36786	14	31174	45
467	478	476	41041	14	31174	46
468	479	471	38556	14	31174	46
469	480	460	38464	14	31174	45
470	481	477	38209	14	31174	46
471	482	463	39360	14	31174	45
472	483	461	39663	14	31174	45
473	484	464	39031	14	31174	45
474	485	465	36913	14	31171	45
475	486	472	39743	14	31174	46
476	487	468	38969	14	31174	46
477	264	480	32783	15	29264	46
478	488	469	37974	14	31174	46
479	489	481	37991	14	31174	46
480	490	482	37959	14	31174	46
481	491	473	38523	14	31174	46
482	492	478	37149	14	31174	46
483	493	474	38780	14	31174	46
484	494	479	38127	14	31174	46
485	495	490	38422	14	31171	46
486	496	483	37055	14	31174	46
487	497	491	38173	14	31174	46
488	498	492	36687	14	31174	46
489	499	484	38894	14	31174	46
490	500	485	37772	14	31174	46
491	258	537	33992	15	29264	47
492	501	486	39701	14	31174	46
493	502	487	36984	14	31174	46
494	503	488	37937	14	31174	46
495	504	493	39253	14	31171	46
496	505	494	39241	14	31174	46
497	506	495	39234	14	31174	46
498	507	504	34978	14	31174	47
499	508	489	39354	14	31174	46
500	509	496	38089	14	31174	46

Appendix F

Fit/Forecasting the Notional C-17 Data Using 15 Observations
and a
Hold-out Sample of 8 Observations
(Work done in EXCEL)

Table F-1 Detailed Summary of Fit/Forecasting the Notional C-17 Data

Forecasting the Notional C-17 Based on a 15 Unit History

Log-Linear

Parameters:	a*	b*
	277.2899795	-0.907621213
SSE(1st 15):	870.72	
SSE(all 23):	1040.10	
Equation:	Y(x)=a*x^b	

Forsythe

Parameters:	a*	b*	cmin*
	1698.48	-2.5006	26.70880797
SSE(1st 15):	291.11		
SSE(all 23):	386.54		
Equation:	Y(x)=a*x^b+cmin		

Stanford-B

Parameters:	a*	beta*	n*
	68.2	-3.1519	-0.3658
SSE(1st 15):	359.22		
SSE(all 23):	364.74		
Equation:	Y(x)=a*(x+beta)^n		

AR(1)

Parameters:	phi*	const*
	0.88464	-0.882
SSE (1st 15):	1266.93	
SSE (all 23):	1727.75	
Equation:	AR(1) = phi*(AR1-1)+const+error	

AR(2)

Parameters:	phi_1*	phi_2*	const*
	0.9010377	-0.0341	-0.253
SSE (1st15):	1340.36		
SSE (all 23):	1753.94		
Equation:	AR(2) = phi_1*(AR2-1)+ phi_2*(AR2-2)+const+error		

AR(3)

Parameters:	phi_1*	phi_2*	phi_3*	const*
	0.88898	-0.00943	-0.03295	0.481052734
SSE (1st 15):	1405.53			
SSE (all 23):	1753.98			
Equation:	AR(3) = phi_1*(AR3-1)+ phi_2*(AR3-2)+phi_3*(AR3-2)+ const+noise			

(more) Table F-1 Detailed Summary of Fit/Forecasting the Notional C-17 Data

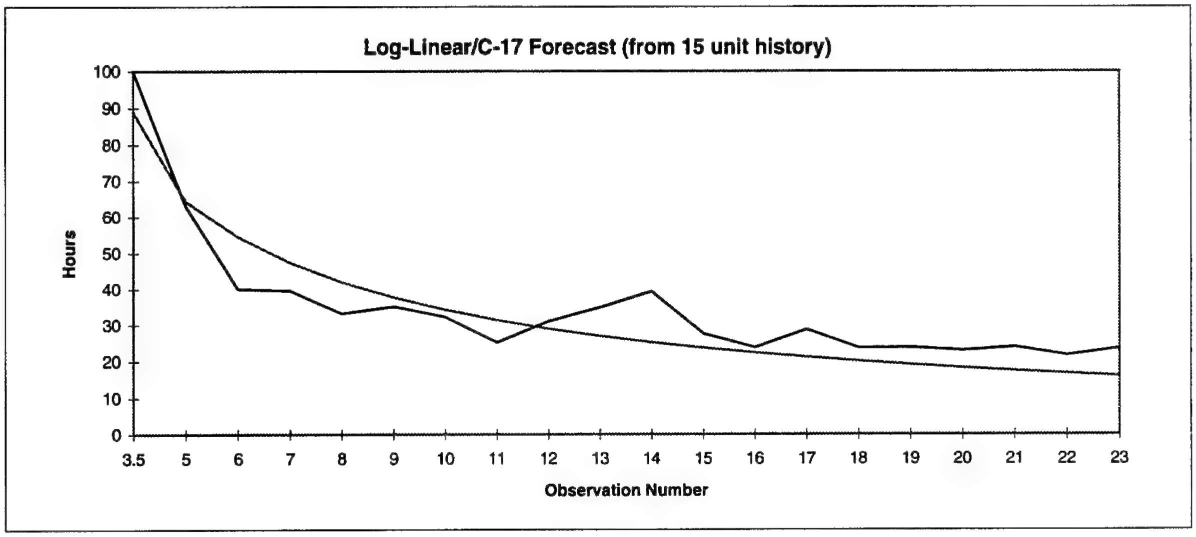
AR(4)					
Parameters:	**phi_1***	**phi_2***	**phi_3***	**phi_4***	**const***
	0.85284	-0.01134	0.02524	-0.08194	2.673475955
SSE (1st 15):	1302.12				
SSE (all 23):	1503.48				
Equation:	**AR(4) = phi_1*(AR4-1)+ phi_2*(AR4-2)+phi_3*(AR4-3)+ phi_4*(AR4-4)+const+noise**				
ARMA(1,1)					
Parameters:	**muprime***	**theta***	**phi***		
	16.9352	1.74432	0.44741		
SSE (1st 15):	89.91				
SSE (all 23):	398.09				
Equation:	**ARMA(1,1)=phi*(ARMA(1,1)-1)+muprime-theta*errorlast+noise**				
ARMA(1,2)					
Parameters:	**muprime***	**phi1***	**theta1***	**theta2***	
	15.9419	0.89942	-0.49149	0.13381	
SSE (1st 15):	3698.99				
SSE (all 23):	10270.02				
Equation:	**ARMA(1,2)=phi1*(ARMA(1,2)-1)+muprime-theta1*errorlast-theta2*errorlastlast+noise**				
ARMA(2,1)					
Parameters:	**muprime***	**phi1***	**phi2***	**theta***	
	21.6784	0.40971	-0.10521	-0.42491	
SSE (1st 15):	329.95				
SSE (all 23):	572.27				
Equation:	**ARMA(2,1)=phi1*(ARMA(2,1)-1)+phi2*(ARMA(2,1)-2)+muprime-theta*errorlast+noise**				
ARMA(2,2)					
Parameters:	**muprime***	**phi1***	**phi2***	**theta1***	**theta2***
	20.4647	0.41473	-0.07662	-0.20164	-0.13734
SSE (1st 15):	302.85				
SSE (all 23):	524.07				
Equation:	**ARMA(2,2)=phi1*(ARMA(2,2)-1)+phi2*(ARMA(2,2)-2)+muprime-theta1*errorlast-theta2*errorlastlast+noise**				

Table F-2 Brief Summary of Fit/Forecasting the Notional C-17 Data

Model	SSE (1st 15)	SSE (all 23)	SSE (last 8)	Rank (1st 15)	Rank (all 23)	Rank (8)
Log-Linear	870.7	1040.1	169.4	6	6	3
Forsythe	291.1	386.5	95.4	2	2	2
Stanford-B	359.2	364.7	5.5	5	1	1
AR(1)	1266.9	1727.7	460.8	7	8	10
AR(2)	1340.4	1753.9	413.6	9	9	9
AR(3)	1405.5	1754.0	348.4	10	10	8
AR(4)	1302.1	1503.5	201.4	8	7	4
ARMA(1,1)	89.9	398.1	308.2	1	3	7
ARMA(1,2)	3699.0	10270.0	6571.0	11	11	11
ARMA(2,1)	329.9	572.3	242.3	4	5	6
ARMA(2,2)	302.9	524.1	221.2	3	4	5

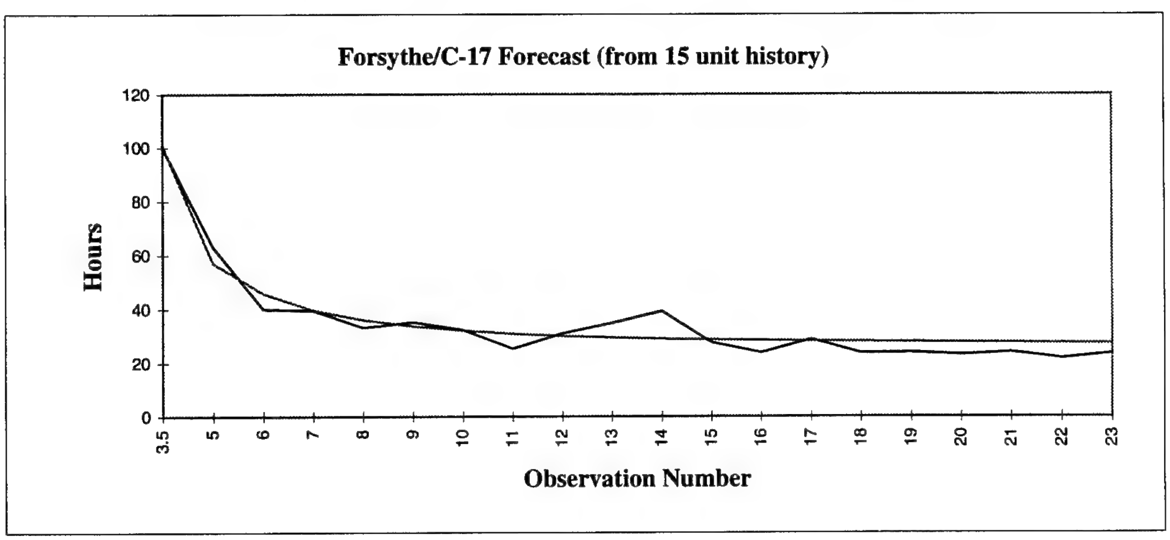
Parameters:	a*	b*
	277.2899795	-0.907621213
SSE(1st 15):	8.707E+02	
SSE(all 23):	1.040E+03	
Equation:	Y(x)=a*x^b	

Figure F-1 EXCEL, Log-Linear Model Fit/Forecast of Notional C-17 Data



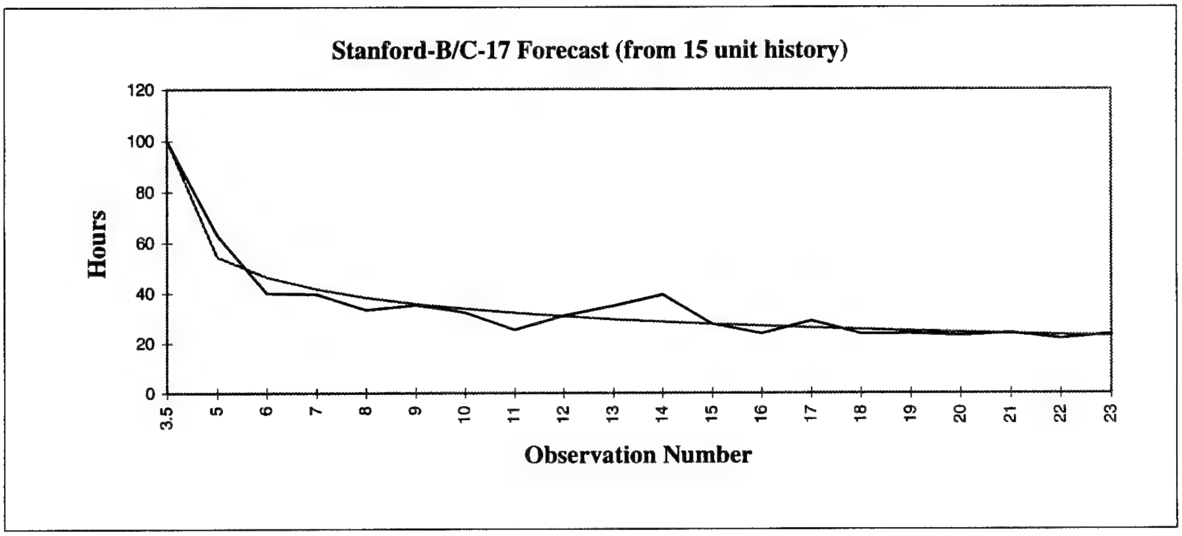
Parameters:	a*	b*	cmin*
	1698.476005	-2.500637253	26.70880797
SSE(1st 15):	2.911E+02		
SSE(all 23):	3.865E+02		
Equation:	Y(x)=a*x^b+cmin		

Figure F-2 EXCEL, Forsythe Fit/Forecast of Notional C-17 Data



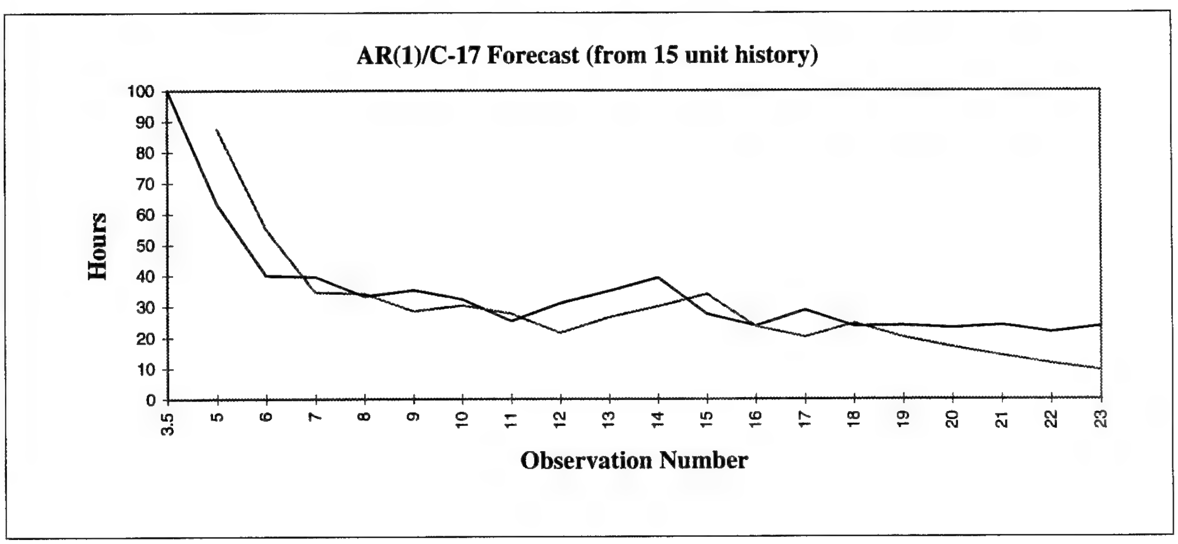
Parameters:	a*	beta*	n*
	68.192195	-3.15186838	-0.365773163
SSE(1st 15):	3.592E+02		
SSE(all 23):	364.7427137		
Equation:	Y(x)=a*(x+beta)^n		

Figure F-3 EXCEL, Stanford-B Fit/Forecast of Notional C-17 Data



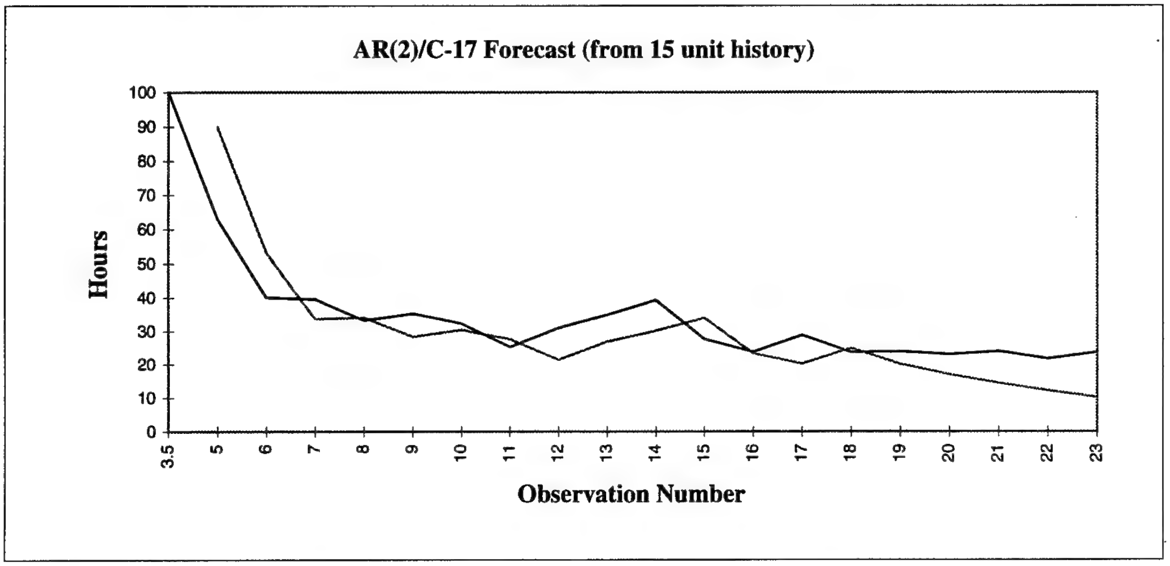
Parameters:	phi*	const*
	0.884635386	-0.882021084
SSE (1st 15):	1.267E+03	
SSE (all 23):	1.728E+03	
Equation:	AR(1) = phi*(Ysubx-1)+const+error	

Figure F-4 EXCEL, AR(1) Fit/Forecast of Notional C-17 Data



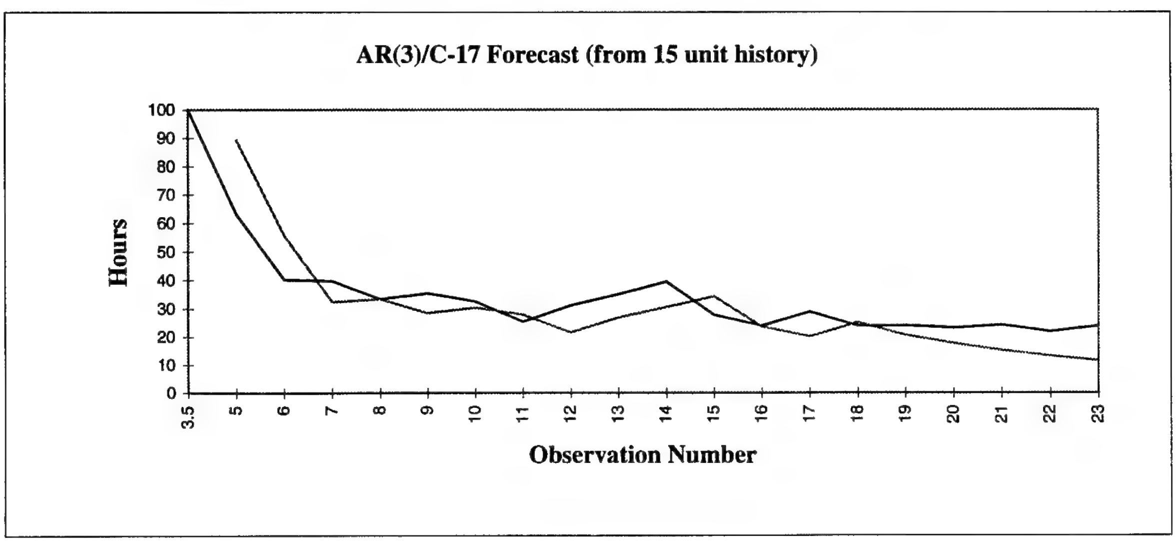
Parameters:	phi_1*	phi_2*	const*
	0.90103771	-0.034061831	-0.252825401
SSE (1st15):	1.340E+03		
SSE (all 23):	1.754E+03		
Equation:	AR(2) = phi_1*(Ysubx-1)+ phi_2*(Ysubx-2)+const+error		

Figure F-5 EXCEL, AR(2) Fit/Forecast of Notional C-17 Data



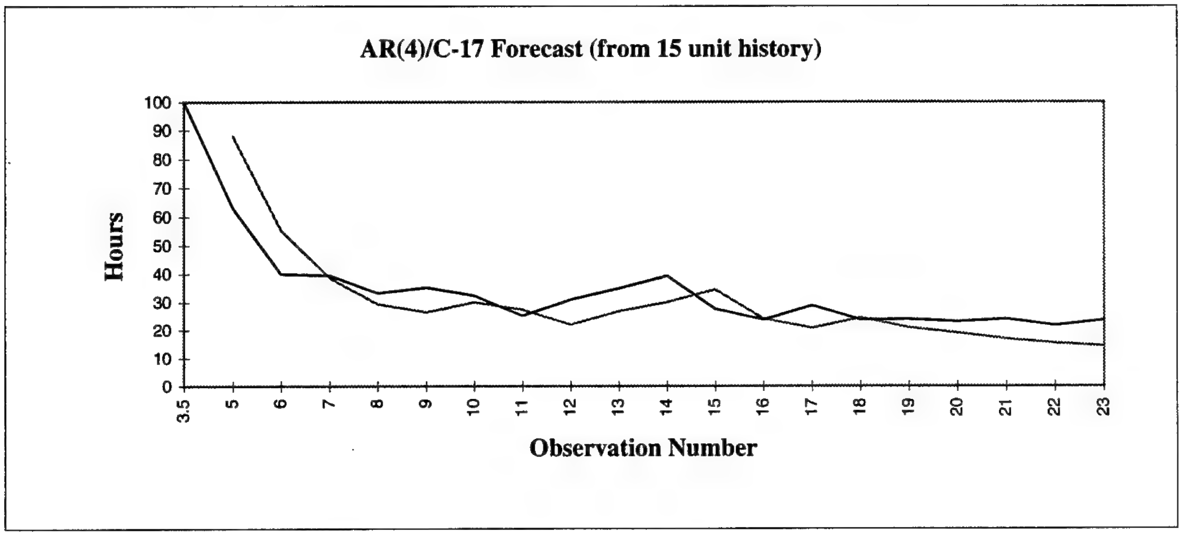
Parameters:	phi_1*	phi_2*	phi_3*	const*
	0.888975471	-0.009425773	-0.032953591	0.481052734
SSE (1st 15):	1.406E+03			
SSE (all 23):	1.754E+03			
Equation:	AR(3) = phi_1*(Ysubx-1)+ phi_2*(Ysubx-2)+phi_3*(Ysubx-2)+const+error			

Figure F-6 EXCEL, AR(3) Fit/Forecast of Notional C-17 Data



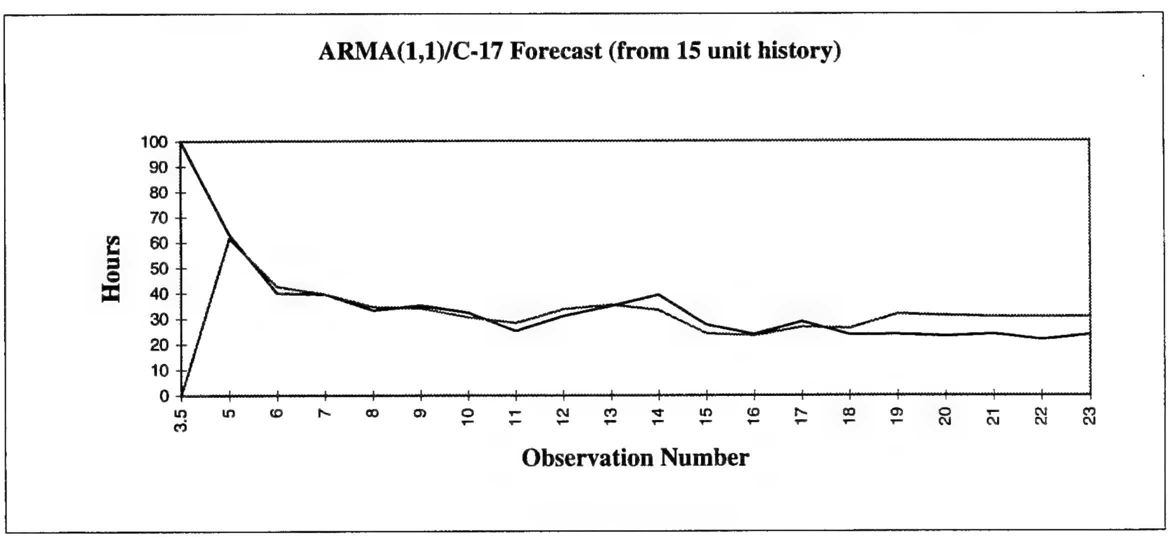
Parameters:	phi_1*	phi_2*	phi_3*	phi_4*	const*
	0.852839607	-0.011344546	0.025242066	-0.081943217	2.673475955
SSE (1st 15):	1.302E+03				
SSE (all 23):	1.503E+03				
Equation:	AR(4) = phi_1*(Ysubx-1)+ phi_2*(Ysubx-2)+phi_3*(Ysubx-2)+const+error				

Figure F-7 EXCEL, AR(4) Fit/Forecast of Notional C-17 Data



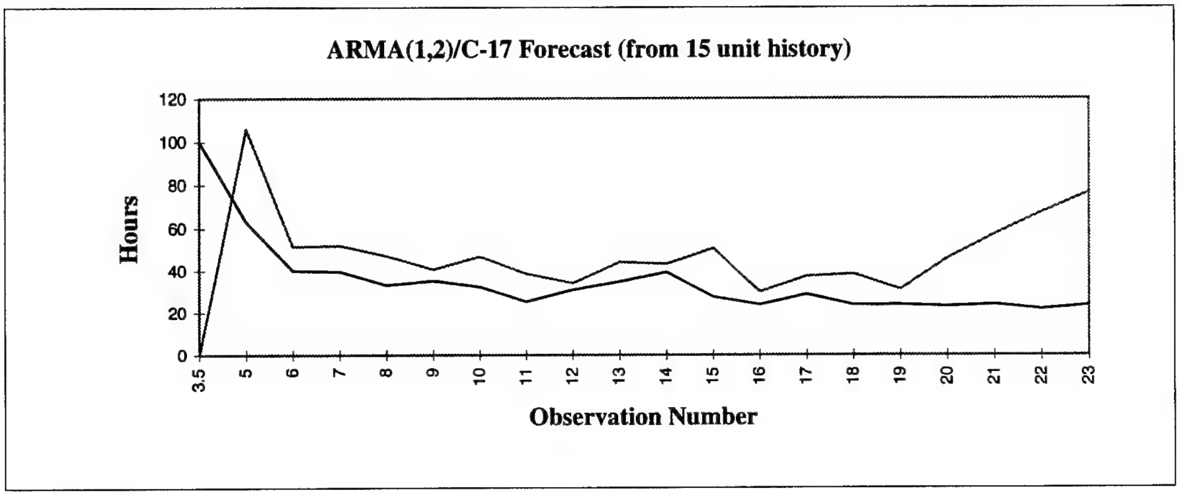
Parameters:	muprime*	theta*	phi*
	16.9352	1.7443	0.4474
SSE (1st 25):	8.991E+01		
SSE (all 500):	3.981E+02		
Equation:	ARMA(1,1)=phi*(ARMA(1,1)-1)+muprime-theta*errorlast+noise		

Figure F-8 EXCEL, ARMA(1,1) Fit/Forecast of Notional C-17 Data



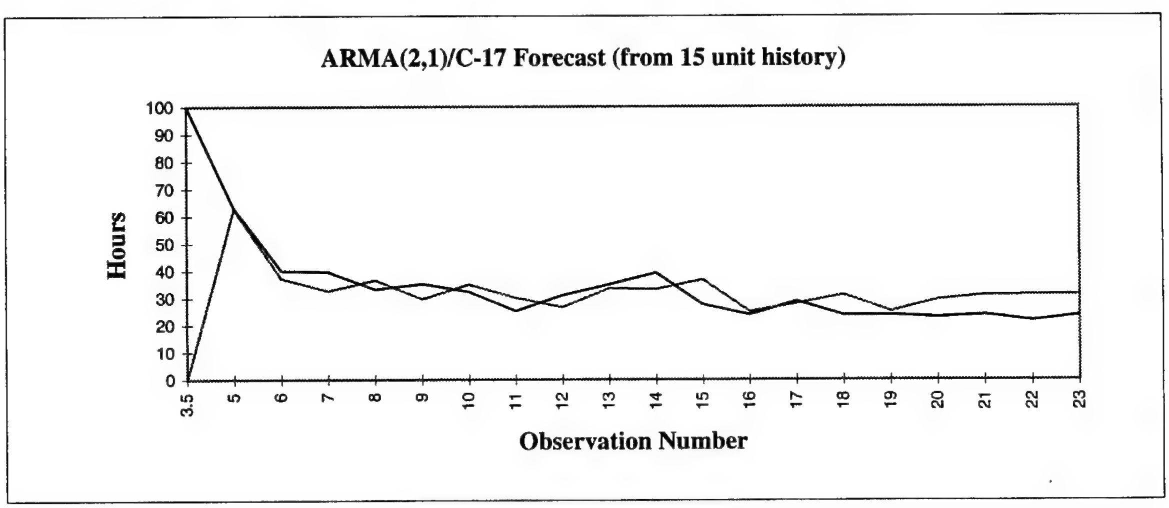
Parameters:	muprime*	phi1*	theta1*	theta2*
	15.9419	0.8994	-0.4915	0.13380841
SSE (1st 15):	3.699E+03			
SSE (all23):	1.027E+04			
Equation:	ARMA(1,2)=phi1*(ARMA(1,2)-1)+muprime-theta1*errorlast-theta2*errorlastlast+noise			

Figure F-9 EXCEL, ARMA(1,2) Fit/Forecast of Notional C-17 Data



Parameters:	muprime*	phi1*	phi2*	theta*
	21.6784	0.4097	-0.105210976	-0.4249
SSE (1st 15):	3.299E+02			
SSE (all 23):	5.723E+02			
Equation:	ARMA(2,1)=phi1*(ARMA(2,1)-1)+phi2*(ARMA(2,1)-2)+muprime-theta*errorlast+noise			

Figure F-10 EXCEL, ARMA(2,1) Fit/Forecast of Notional C-17 Data



Parameters:	muprime*	phi1*	phi2*	theta1*	theta2*
	20.4647	0.4147	-0.0766	-0.201639264	-0.137342779
SSE (1st 15):	3.029E+02				
SSE (all 23):	5.241E+02				
Equation:	ARMA(2,2)=phi1*(ARMA(2,2)-1)+phi2*(ARMA(2,2)-2)+ muprime-theta1*errorlast-theta2*errorlastlast+noise				

Figure F-11 EXCEL, ARMA(2,2) Fit/Forecast of Notional C-17 Data

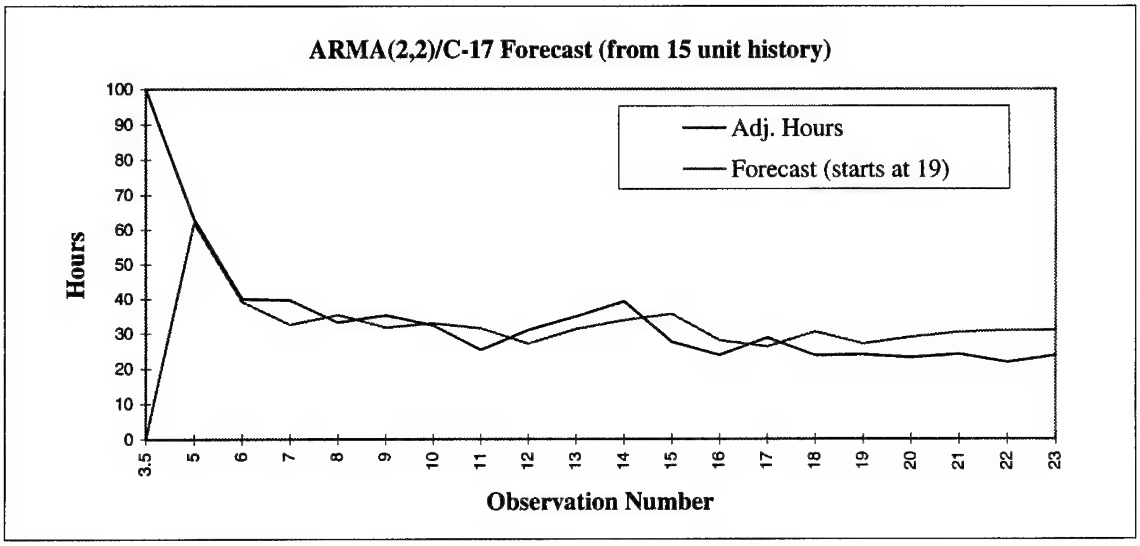


Table F-3 The Notional C-17 Data Base

Unit Number	Adj. Hours
3.5	100
5	63.04729214
6	40.06102212
7	39.63005339
8	33.24942792
9	35.25934401
10	32.31121281
11	25.27841342
12	31.03928299
13	34.94279176
14	39.33638444
15	27.60869565
16	23.78718535
17	28.80244088
18	23.69946606
19	23.85964912
20	23.07398932
21	24.00839054
22	21.73150267
23	23.67276888

VITA

Captain Jennie H. Lommel was born on 26 June 1958 in Manhasset, New York. After graduating from Oyster Bay High School in Oyster Bay, New York in 1976, she worked for a couple of years before marrying her husband Walter Lommel, also of Oyster Bay. They both enlisted in the United States Air Force early in 1979.

Upon graduation as an honor graduate from Electronic Warfare Systems School school in 1980, she was assigned to Eglin AFB as an EW technician. Upon completion of her assignment at Eglin, she volunteered for instructor duty at the EW school at Keesler AFB, Mississippi. During both of these assignments, Jennie took classes towards the completion of her degree in Electrical Engineering.

In 1986, she was selected to participate in the Airman's Education and Commissioning Program (AECP) and went off to complete her BSEE at Arizona State University. After graduation from ASU and from Officer Training School, Jennie was commissioned a second lieutenant in the United States Air Force. After her commission, Lt Lommel's first assignment was to Aeronautical Systems Division (ASD), Wright-Patterson AFB, Ohio, where she was assigned as an analyst in the Aeronautical Observables Branch. During this time, Lt Lommel also acted as the executive officer to the director of the Avionics Systems Directorate.

Captain Lommel entered the School of Engineering, Air Force Institute of Technology, Wright-Patterson Air Force Base, Ohio in 1994. After graduating with a Master of Science degree in Operations Research, she was assigned NAIC at Wright-Patterson AFB, Ohio.

Captain Lommel has two children: a daughter, Kate; and a son, Alex.

Captain Jennie H. Lommel
85 Melbourne Street
Oyster Bay, New York 11771

Epilogue

The Data -- A Humorous Aside

As an entertaining aside which may actually not survive the editorial shears of my advisor, I've got to say that searching for a data base on which to test the aforedeveloped models turned out to be quite the formidable and frustrating task! In the final frantic throws of my thesis process (and under penalty of not graduating....), I searched high and low for this data. My advisor searched high and low too and even went so far as to call in some personal debts -- all to no avail (???).

My search led me to an old friend in the Defensive Avionics System Program Office (SPO) here on base. He pointed me towards, what he termed, the 'manufacturing pukes' (MPs) in the aircraft SPOs. One after another, the MPs suggested, "Sorry, we can't help you. You'll need to go directly to the manufacturer to get *that* kind of data! Be forewarned, however, they might consider it proprietary and opt not to give it to you."

So, that suggestion fresh in my mind, I went to the Aeronautical Systems Center (ASC) Technical Library to see what they had before attempting an assault on the manufacturers. When the people at the Tech Library heard what I wanted, they did the proverbial, "You want it WHEN????!!!" Then they told me, "Sorry, we can't help you. You'll need to go directly to the manufacturers to get *that* kind of data. Be forewarned, however, they might consider it proprietary and opt not to give it to you." (Sound familiar?)

Okay, manufacturers, here I come! Upon questioning, the one fellow I *did* manage to get in contact with at an un-named aircraft manufacturing company (my

middle name is chicken or I'd name the company as well as the individual) said, "I can't help you; *that* data is proprietary. You'd do best to talk to the people at the SPOs." Geee Wizzz! I think I'm on my own.

This tale of woe does have a good as well as an entertaining ending. In my final search for a lead and one last attempt to secure my graduation, I spoke to the Operations Research Department Head, Lieutenant Colonel Paul Auclair. He promptly pointed me towards the Logistic School (another school at AFIT) to a man named Dr. Vaughn (AFIT Department of Research), who promptly pointed me towards Professor Roland Kankey (AFIT Department of Acquisition) who promptly pointed me (and quite accurately so!) to the ASC Cost Library where I found the data for which I'd been so fervently searching.

I left the library with a plethora of publications with dates ranging from 1949 to 1982. The study dated 1982 and entitled The Learning Curve in the Airframe Industry (Brewer, 1982) is the source of my actual aircraft build history data -- the 1000 unit history of the F-102. Before I discovered my wonderful data base, and while leafing through the stack of documents, I found an interesting comment in the work entitled An Airframe Production Function (Alchian, 1949):

> "Within individual airframe manufacturing facilities, estimates of costs of producing airplanes are frequently based on the learning curve. Unfortunately, no data on the estimates made by aircraft manufacturing facilities themselves have been available, so tests of their reliability could not be made. Naturally enough, individual business units are not willing to publicly reveal their cost estimating records."

I guess maybe he didn't graduate on time.

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In 1995, the C-17 Factory Simulation Model (FSM) was developed to enable analysts to address "what-if" questions about the resources required to build future aircraft, and is based on learning curve models that are used to both portray and simulate future aircraft production. In this thesis, we examine and develop alternate learning curve models that also utilize a small amount of initial production data to portray the relationship between the number of aircraft built and the resources required to build them. The goal is to identify a model which not only provides a good fit and forecast based on a small amount of data but is also intuitive and reasonably simple to apply. We also propose and evaluate the use of Autoregressive Moving Average (ARMA) models for modeling the effects of learning. These models are exercised in fitting simulated log-linear data, as well as in fitting and forecasting historical F-102 manufacturing data and notional C-17 manufacturing data. The results are somewhat inconclusive since they do not identify any one model as the best. They do, however, suggest that ARMA models are a promising alternative to the standard log-linear learning curve. The thesis concludes with an examination of the effects of explicitly accounting for uncertainty in parameter estimation when simulating future performance based on the traditional log-linear learning curve model. The results show that the approach employed in the FSM is viable even though it does not directly account for this uncertainty.

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